Proper Orthogonal Decomposition and Wavelet Analysis of Sloshing Flows

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Overview

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Material and Methods

- Data processing strategies
- Experimental setup
- Test cases

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- Data prequalification
- Acceleration wavelet transform
- Wavelet–based damping
- Mathematical model
- Proper Orthogonal Decomposition & Wavelet
- Wavelet Coherency

Objective

Sloshing and **structural dynamics** coupling is important in the design phase, modeling, and control of several aerospace systems: rockets, satellites, and aircrafts.

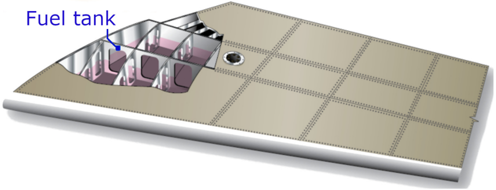


Figure: Example of coupling between sloshing and structural dynamics: an integral tank within a wing.

The aim of this work is to develop a **data analysis technique** able to highlight the interaction between fluid and structure.

Data processing strategy (1)

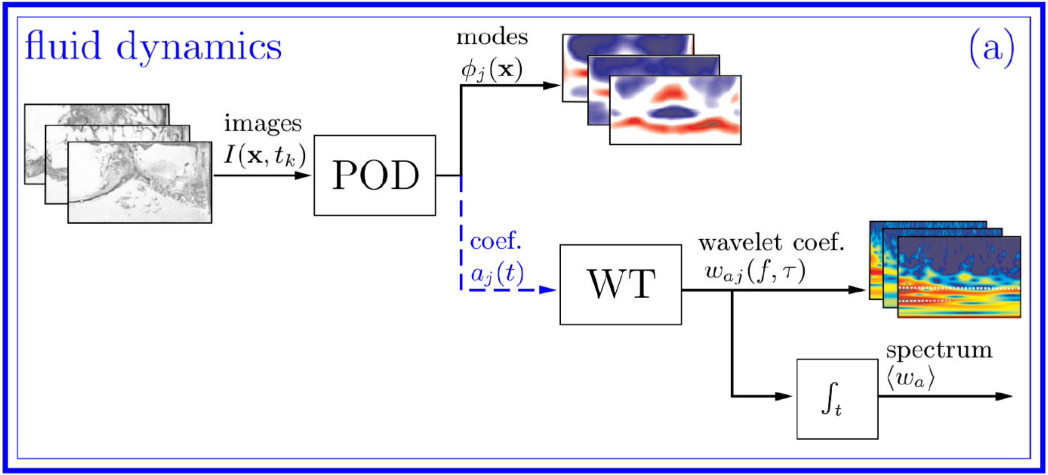
The main idea is to **combine Proper Orthogonal Decomposition** (POD) and **Wavelet Transform** (WT) to analyse an image time-resolved sequence.

$$I(\boldsymbol{x}, t) \approx \sum_{i=1}^{N} a_i(t)\phi_i(\boldsymbol{x}) \approx \sum_{i=1}^{N}\sum_{j=1}^{M} a_{ij}^{\psi}(t)\phi_i(\boldsymbol{x}) \tag{1}$$

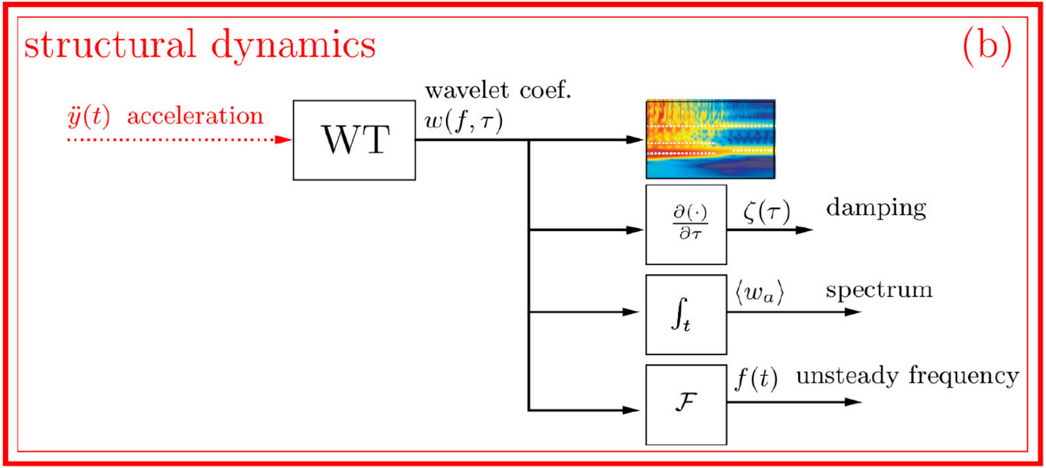
Decomposition characteristics:

- $\phi_i(\boldsymbol{x})$ coherent in space
- $a_{ij}^{\psi}(t)$ coherent in time

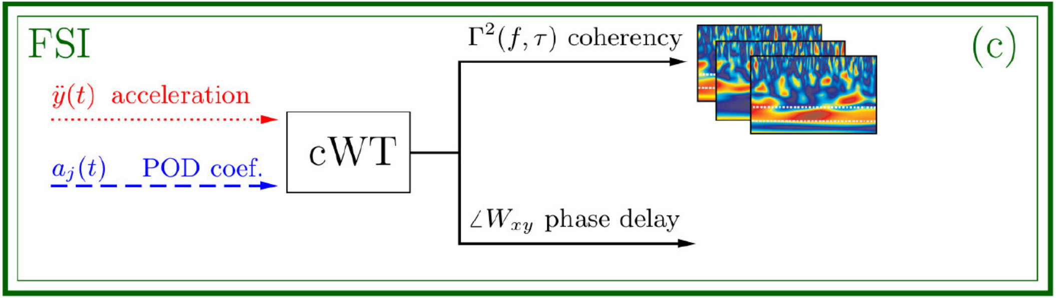
Data processing strategy (2)



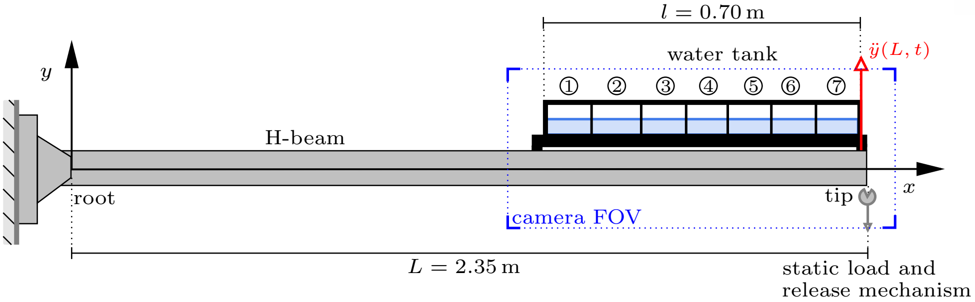
Data processing strategy (3)



Data processing strategy (4)



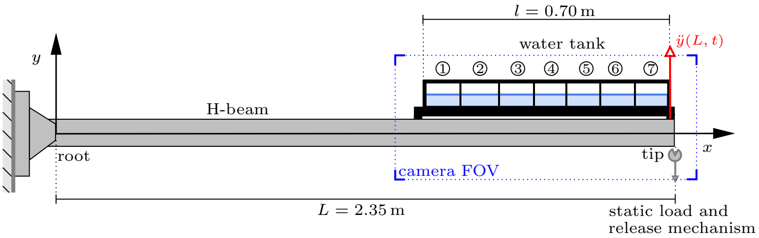
Experimental setup



Instrumentation and acquisition details:

- Accelerometer – isotron model 65L-100, $S = 102.7 mV/g$, $f_s^{\ddot{y}} = 1\,kHz, T^{\ddot{y}} = 6\,s$
- Fast camera – photron fastcam SA1.1, $f_s^I = 3kHz, T^I = 10\,s$

Test cases



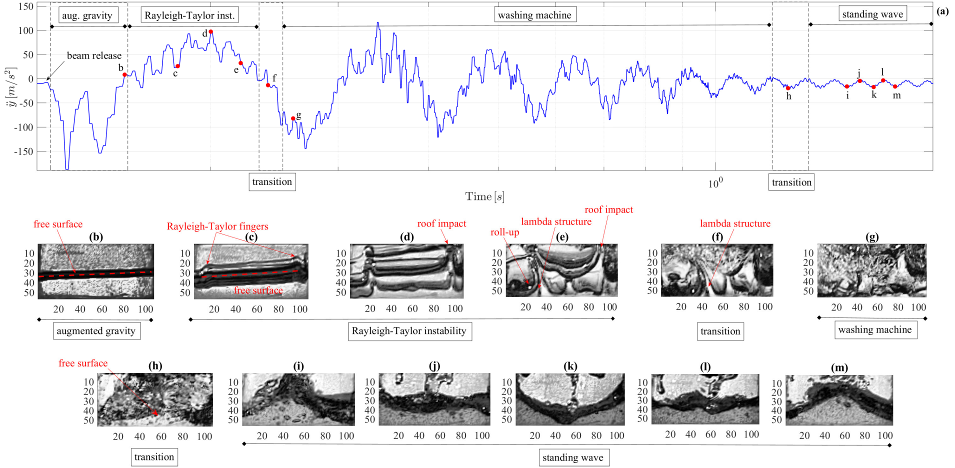
Water volume ratio:

$$\alpha = \frac{V_w}{V_{tot}}$$

Test Cases:

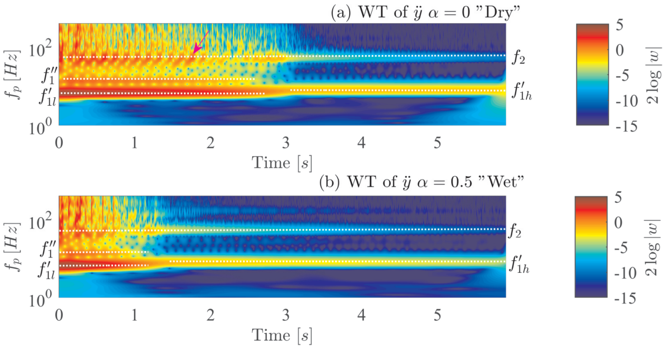
- Test case 1: $\alpha = 0$, dry;
- Test case 2: $\alpha = 0.3$, wet;
- Test case 3: $\alpha = 0.5$, wet.

Data prequalification



Wavelet Transform of the Acceleration

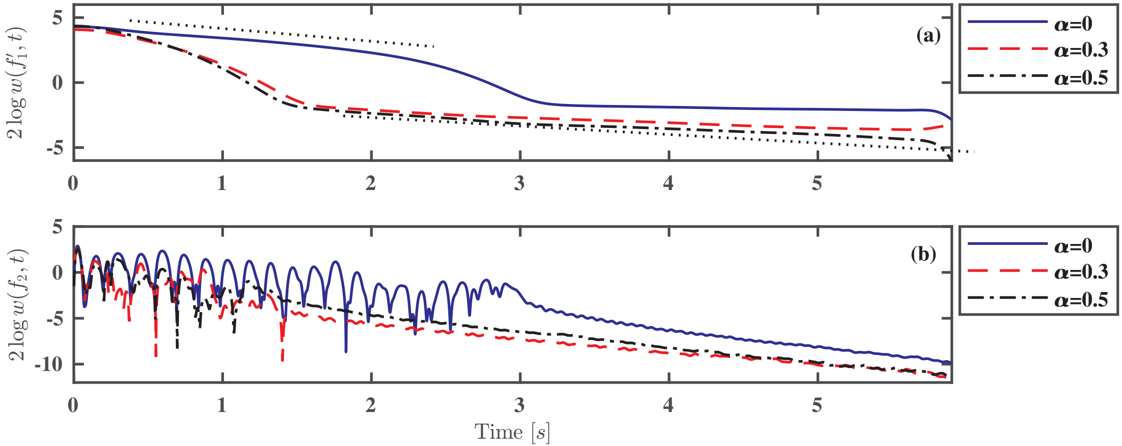
Mode	Structural dynamics freq. [Hz]	Description
f'_{1l}	7.4	1^{st} mode and 1^{st} harmonic
f'_{1h}	7.9	1^{st} mode and 1^{st} harmonic
f''_{1}	13.9	1^{st} mode 2^{nd} harmonic
f_2	50.2	2^{nd} mode



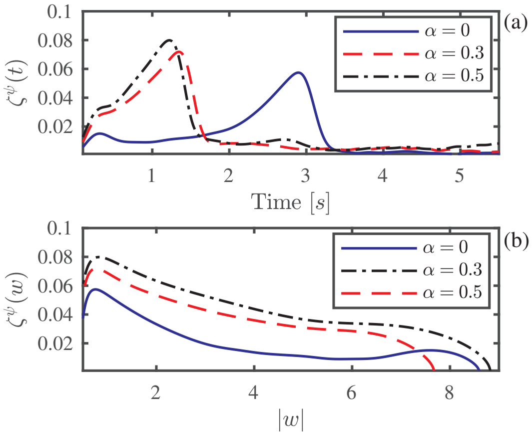
H–Beam modes characteristics:

- First mode is affected by an abrupt drift in frequency from 7.4 to 7.9 Hz;
- Second mode has the frequency weakly time dependent.

Wavelet Coefficients of the Acceleration



Wavelet–Based Damping



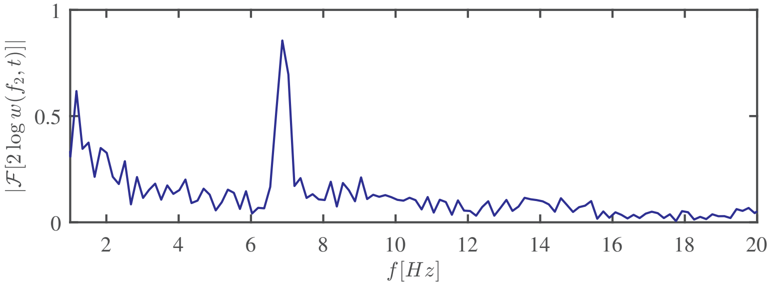
Wavelet–based damping[1]:

$$\zeta^{\psi}(\tau) = -\frac{1}{\omega_n}\frac{\partial \ln |w(f_1', \tau)|}{\partial \tau} \qquad (2)$$

- Test case 2–3: damping is higher and appears before than case 1 (see fig.a)
- Test case 2–3: damping is higher for all acceleration values (see fig.b)
- Damping is a function of the acceleration $\rightarrow \zeta = f(\ddot{y})$

[1] Chen *et al.*, Wavelet analysis for identification of damping ratios and natural frequencies, J. Sound Vib., 323(1-2), 130–147.

Fourier Transfrom of the Wavelet Coefficients



The **frequency of the second mode oscillates** with a frequency f_1', equal to the frequency of the first mode:

$$f_2(t) = \langle f_2(t) \rangle + c \sin\left(2\pi f_1' t\right) \tag{3}$$

Simple Mathematical Model

From the experimental evidences, we have a preliminary idea of the mathematical model:

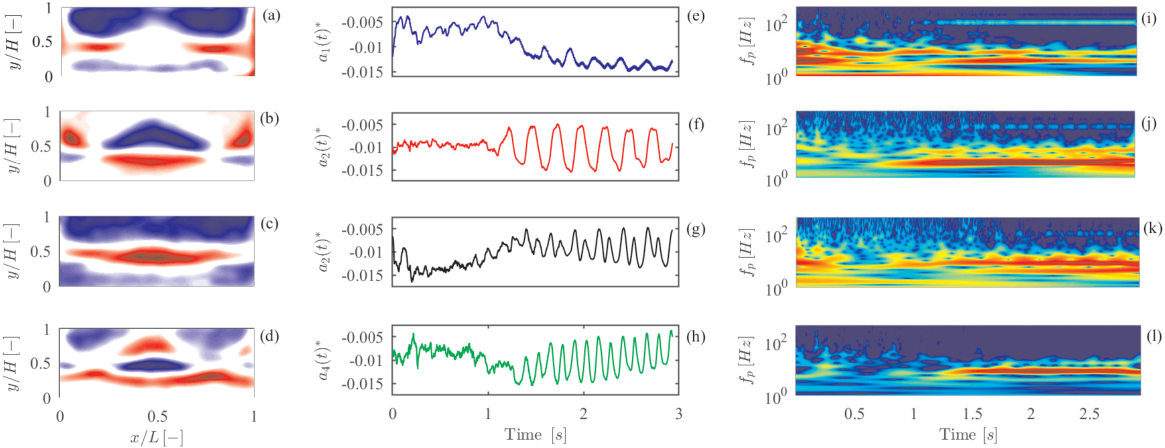
- for the first mode

$$\ddot{y} + 2\zeta(\ddot{y})\omega_1\dot{y} + \omega_1^2 y = 0 \tag{4}$$

- for the second mode

$$\ddot{y} + 2\zeta(\ddot{y})\omega_2(t)\dot{y} + \omega_2^2(t) y = 0 \tag{5}$$

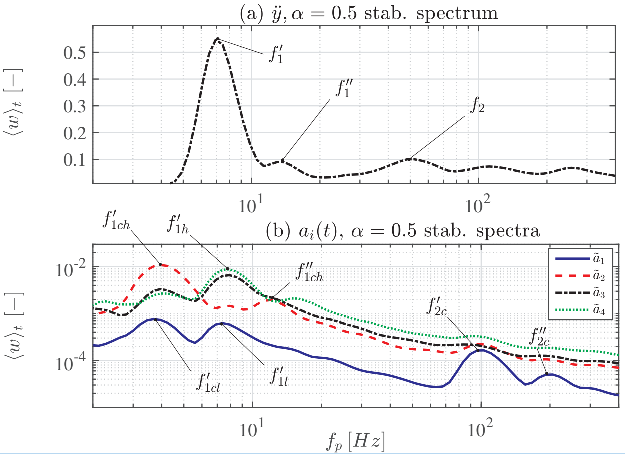
Proper Orthogonal Decomposition & Wavelet



Test case 3: stabilised Spectra

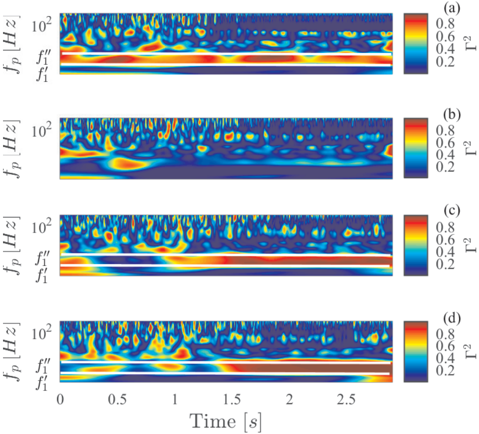
Mode	Structural dynamics freq. $[Hz]$	Description
f'_{1l}	7.4	1^{st} mode–1^{st} harmonic
f'_{1h}	7.9	1^{st} mode– 1^{st} harmonic
f''_{1}	13.9	1^{st} mode– 2^{nd} harmonic
f_2	50.2	2^{nd} mode

Mode	Fluid dynamics freq. $[Hz]$	Description
f'_{1l}	7.4	Coupling between ϕ_1 and $\ddot{y}$
f'_{1h}	7.9	Coupling between ϕ_3-ϕ_4 and $\ddot{y}$
f'_{1cl}	3.7	symmetric sloshing
f'_{1ch}	4.0	symmetric sloshing



Wavelet Coherency

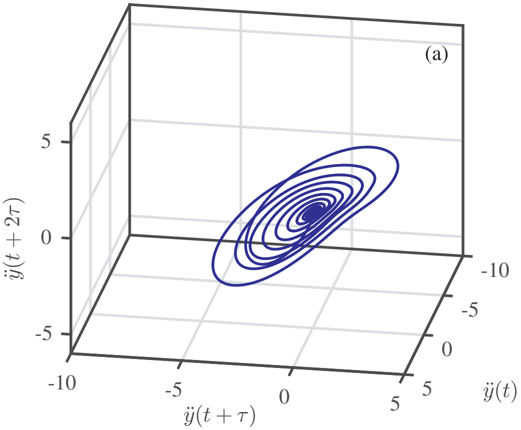
Mode	Fluid dynamics frequency $[Hz]$	Description
f'_{1l}	7.4	Coupling ϕ_1 and $\ddot{y}$ (all the time)
f'_{1h}	7.9	Coupling ϕ_3 and ϕ_4 (coherent regime)
f'_{1cl}	3.7	symmetric sloshing (no coupling)
f'_{1ch}	4.0	No coupling ϕ_2 and $\ddot{y}$

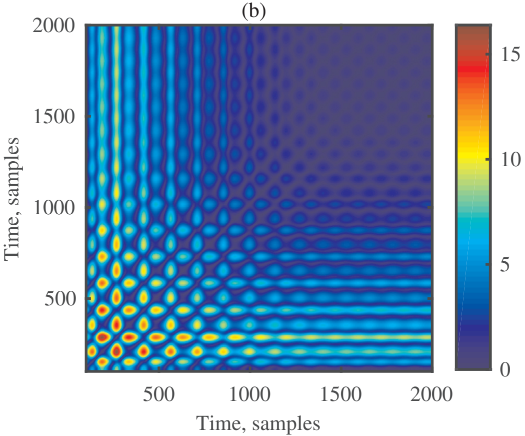


Summary & Conclusions

1. A novel technique to decompose the hydrodynamic flow field in coherent modes in time and space (a_{ij}^{ψ} and ϕ_i) is developed and tested on an experimental database.
2. A wavelet–based damping ratio calculation is applied and it appears to be promising.
3. Water contributes to increase the damping ratio.
4. The frequency of the second mode of the beam is not constant in time and oscillates with the same frequency as the first mode.
5. A simple mathematical model for the first and second mode of the beam is proposed
6. Strong coupling between structural dynamics and hydrodynamics is detected:
   - ϕ_1 oscillates at the same frequency as the beam for all the time;
   - ϕ_3 and ϕ_4 oscillate at the same frequency as the beam only during the coherent regime;
   - ϕ_2 no linear coupling with acceleration.

Future Work


(a)
5
0
-5
-10
-5
0
5
$\ddot{y}(t+2\tau)$
$\ddot{y}(t+\tau)$
-10
-5
0
5
$\ddot{y}(t)$


(b)
2000
1500
1000
500
Time, samples
500
1000
1500
2000
Time, samples
15
10
5
0

Thank you for your Attention