Advances in Reduced Order Modelling for Linear and Nonlinear Sloshing

Marco Pizzoli, Francesco Saltari, Franco Mastroddi

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.

Contents of presentation

- **Reduced Order Models for sloshing fluids** into aircraft tanks are a challenging issue for aircraft design and controls: this is one of the targets within SLOWD H2020 project.

- **Linear ROMs** for sloshing fluid into aircraft tank are developed so allowing a high-fidelity coupling with aircraft aeroelasticity and flight dynamics within the framework of acceptance of linear hypothesis (lateral sloshing).

  An actual application on a flexible aircraft is presented.

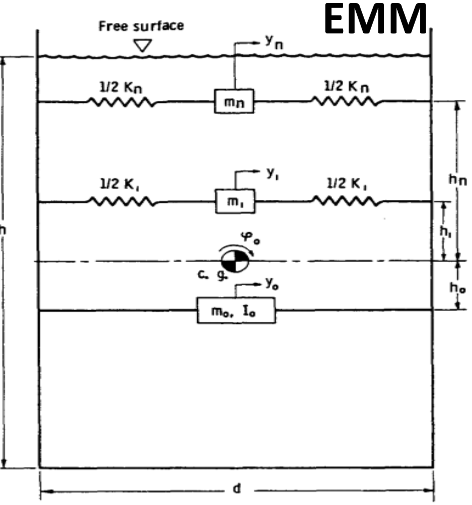
- **Nonlinear ROM** are related to vertical sloshing which includes complex phenomena as Rayleigh–Taylor instability.
  - ✓ A **metric** for the characterization and quantification of the *nonlinear damping* introduced by vertical sloshing
  - ✓ Development of a simplified model of **bouncing ball** inside tank that replaces time-consuming CFD simulations in order to generate data
  - ✓ **Neural-Network-based ROM** for perspective integration of vertical sloshing into aeroelastic framework

Linear ROM for sloshing

Equivalent Mechanical Models (EMMs)

A physical motivation for using them as linear ROMs (LD)

EMM



By imposing a lateral **rigid** motion $\tilde{Y}$ and a pitch motion $\tilde{\varphi}^{(G)}$ one can obtain **force** $\tilde{F}_y$ and $\tilde{M}_x^{(G)}$ **moment** (FD)

$$\tilde{F}_y = -s^2 m_T \left[1+\sum_{n=1}^{\infty}\frac{m_n}{m_T}\left[\frac{\frac{-s^2}{\omega_n^2}}{1+\frac{s^2}{\omega_n^2}}\right]\right]\tilde{Y} - m_T s^2 \sum_{n=1}^{\infty}\frac{m_n}{m_T}\left[\frac{-h_n\frac{s^2}{\omega_n^2}+\frac{g}{\omega_n^2}}{1+\frac{s^2}{\omega_n^2}}\right]\tilde{\varphi}^{(G)}$$

$$\tilde{M}_x^{(G)} = -s^2\sum_{n=1}^{\infty} m_n\left[\frac{-h_n\frac{s^2}{\omega_n^2}+\frac{g}{\omega_n^2}}{1+\frac{s^2}{\omega_n^2}}\right]\tilde{Y} - s^2\left[I_0 + m_0 h_0^2 + \sum_{n=1}^{\infty} m_n h_n^2 + \sum_{n=1}^{\infty} m_n\left[\frac{-h_n^2\frac{s^2}{\omega_n^2}+\frac{g^2}{\omega_n^2-s^2}+\frac{2h_n g}{\omega_n^2}}{1+\frac{s^2}{\omega_n^2}}\right]\right]\tilde{\varphi}^{(G)}$$

mass spring system **properly distributed** in order to be identified with the physical quantities given by the **potential theory**

$$\begin{Bmatrix}\tilde{F}_y\\ \tilde{M}_x^{(G)}\end{Bmatrix} = \begin{bmatrix}G_{YY}(s) & G_{Y\varphi}(s)\\ G_{\varphi Y}(s) & G_{\varphi\varphi}(s)\end{bmatrix}\begin{Bmatrix}\tilde{Y}\\ \tilde{\varphi}^{(G)}\end{Bmatrix}$$

$$\mathsf{G}(s) = s^2\mathsf{A}^{\mathsf{A}} + \left(s^2\mathsf{B}^{\mathsf{A}} + \mathsf{C}^{\mathsf{A}}\right)\left(s^2\mathsf{I} + \Omega^{\mathsf{A}^2}\right)^{-1}\left(s^2\mathsf{B}^{\mathsf{A}} + \mathsf{C}^{\mathsf{A}}\right)^{\mathsf{T}}$$

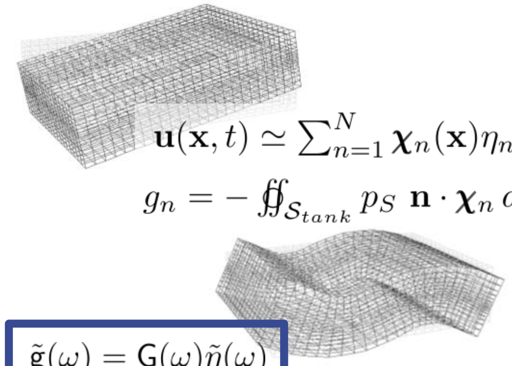
Frequency Response Function Matrix

Evaluation of a general sloshing operator via O/I identificaton technique

Possibility to synthesize this operator in frequency domain by exploiting **CFD transient simulations** (or experiments)

- **3D motion** (including deformations)
- any **tank shape**

1° STEP: Definition of a set of generalized **shape functions**

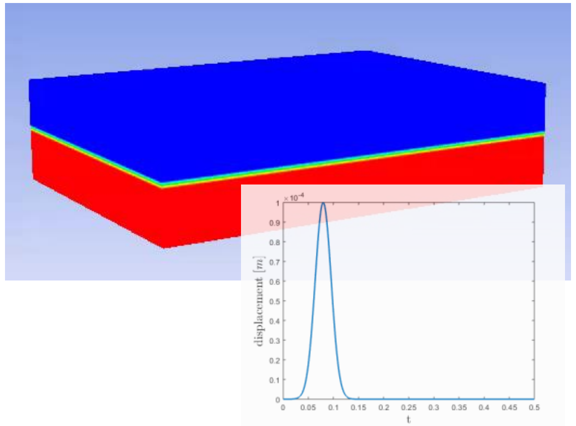


$$\mathbf{u}(\mathbf{x},t) \simeq \sum_{n=1}^{N} \boldsymbol{\chi}_n(\mathbf{x})\eta_n(t)$$

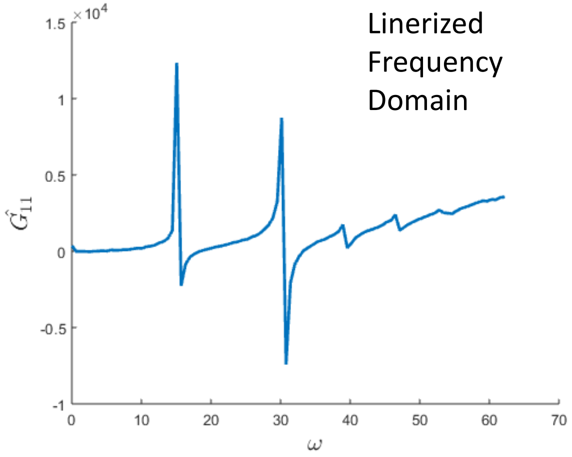
$$g_n = -\oiint_{S_{tank}} p_S\, \mathbf{n}\cdot\boldsymbol{\chi}_n\, dS$$

$$\tilde{\mathsf{g}}(\omega) = \mathsf{G}(\omega)\tilde{\eta}(\omega)$$

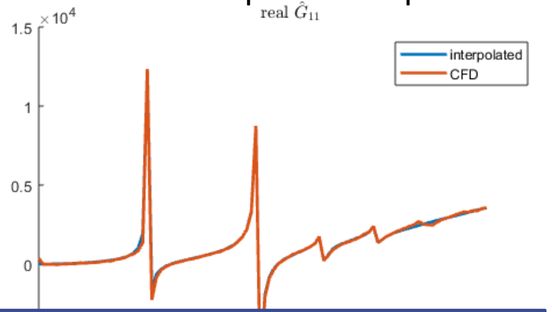
2° STEP: Perform as many simulation as the number of gen. Shape functions



3° STEP: obtain $G_{ij}(\omega)$ via O/I ratio



4° STEP: «curve fitting» based on the EMM inspired interpolation



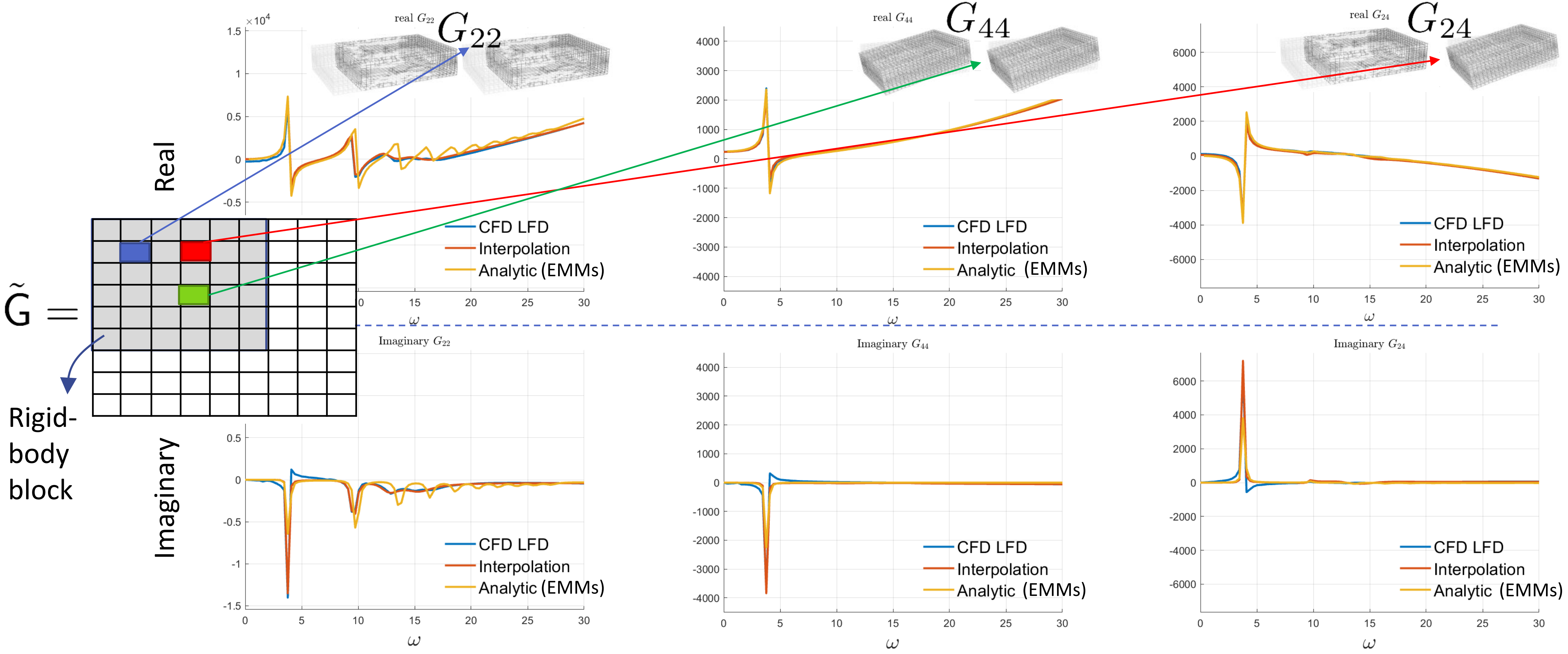
Time domain

$$\mathsf{g} = \bar{\mathsf{A}}\ddot{\eta} + \bar{\mathsf{B}}\ddot{\mathsf{r}} + \bar{\mathsf{C}}\mathsf{r}$$

$$\ddot{\mathsf{r}} + 2\bar{\Gamma}\,\bar{\Omega}\,\dot{\mathsf{r}} + \bar{\Omega}^2\,\mathsf{r} = \bar{\mathsf{B}}^{\mathsf{T}}\,\ddot{\eta} + \bar{\mathsf{C}}^{\mathsf{T}}\,\eta$$

Identified sloshing operator

Comparison between EMMs and identified ROM for tank modes



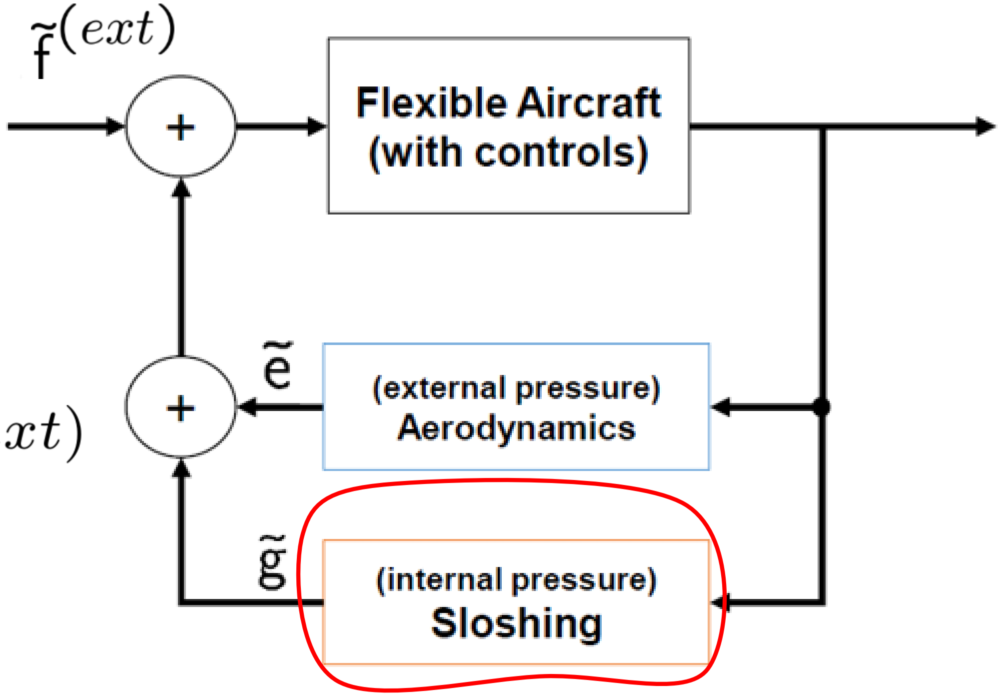
Final integration of sloshing ROM into flexible aircraft aeroelastic formulation

$$
\begin{aligned}
\bar{\mathsf{g}}^{(i)} &= \mathsf{R}^{(i)}\mathsf{g}^{(i)} \\
\eta^{(i)} &= \mathsf{R}^{(i)\mathsf{T}}\mathsf{q}^{(i)}
\end{aligned}
$$

Time domain

$$\mathsf{M}\ddot{\mathsf{q}} + \mathsf{K}\mathsf{q} = \mathsf{e}(\mathsf{q}) + \bar{\mathsf{g}}(\mathsf{q}) + \mathsf{f}^{(ext)}$$

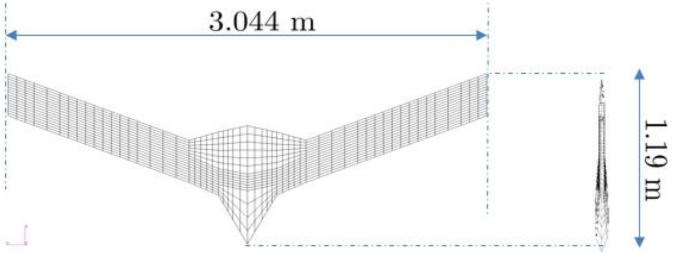
Aerodynamic forces   Sloshing forces

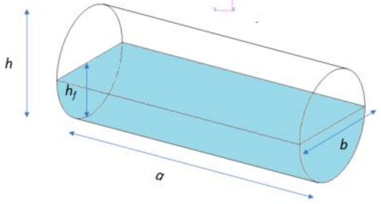


Frequency domain

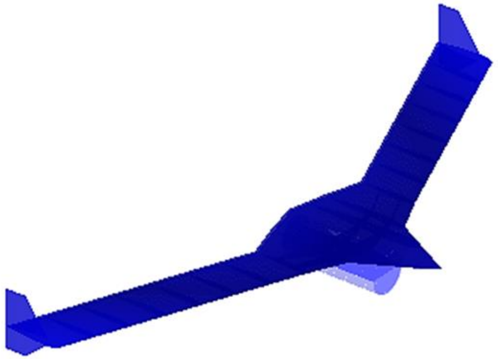
$$s^2\mathsf{M}\tilde{\mathsf{q}} + \mathsf{K}\tilde{\mathsf{q}} = q_D\mathsf{E}(s)\tilde{\mathsf{q}} + \mathsf{R}\,\mathsf{G}(s)\mathsf{R}^\mathsf{T}\tilde{\mathsf{q}} + \tilde{\mathsf{f}}^{(ext)}$$

The flying wing test case



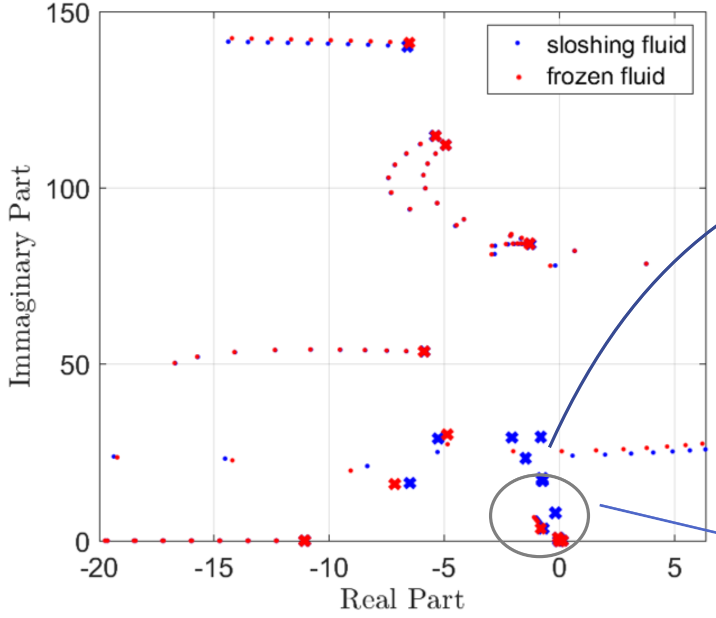


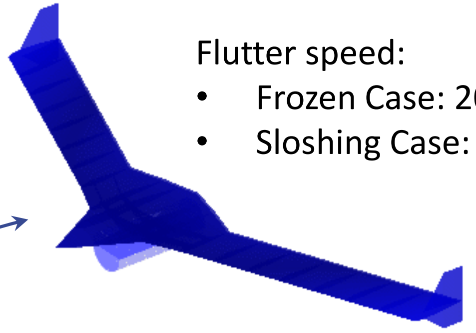
The tank is a rigid horizontal cylinder



The mass of the fluid is 20% of the overall structure mass

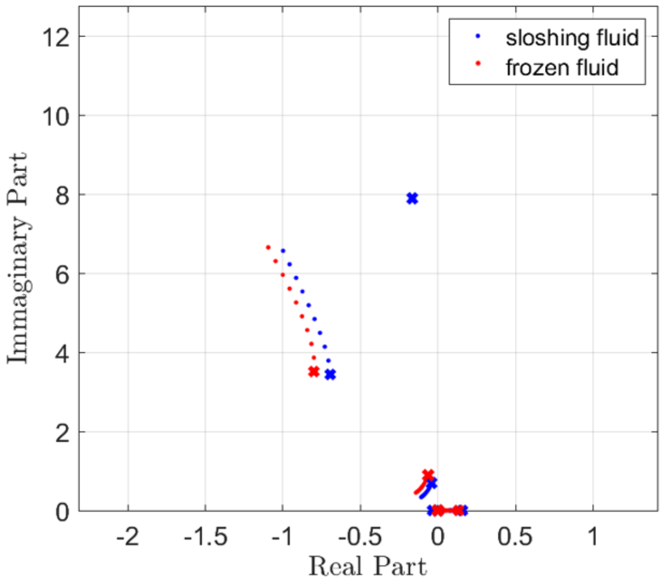
Flight speed: 15 m/s → 30 m/s





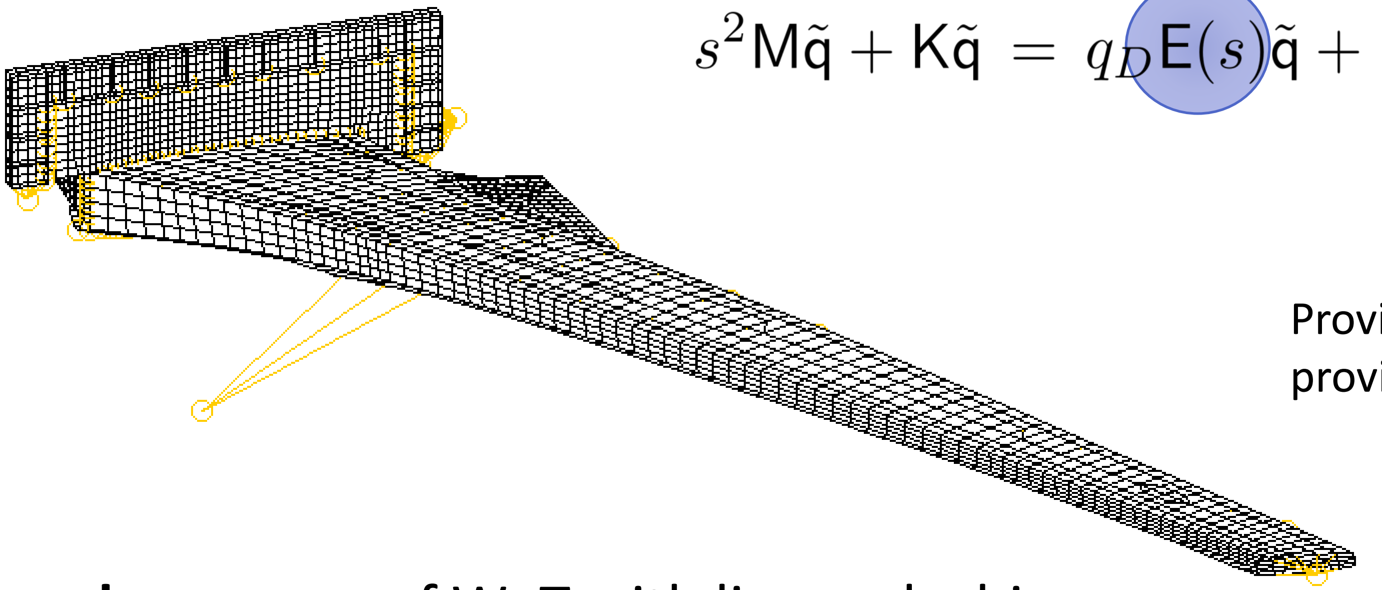
Flutter speed:

- Frozen Case: 20 m/s
- Sloshing Case: 19.4 m/s



Next step?

AIRBUS Wing of Tomorrow (WoT)



$$s^2\mathsf{M}\tilde{\mathsf{q}} + \mathsf{K}\tilde{\mathsf{q}} = q_D\mathsf{E}(s)\tilde{\mathsf{q}} + \mathsf{R}\,\mathsf{G}(s)\mathsf{R}^\mathsf{T}\tilde{\mathsf{q}} + \tilde{\mathsf{f}}^{(ext)}$$

Provided by CFD simulations provided by SLOWD partners

Aeroelastic stability and response of WoT with linear sloshing

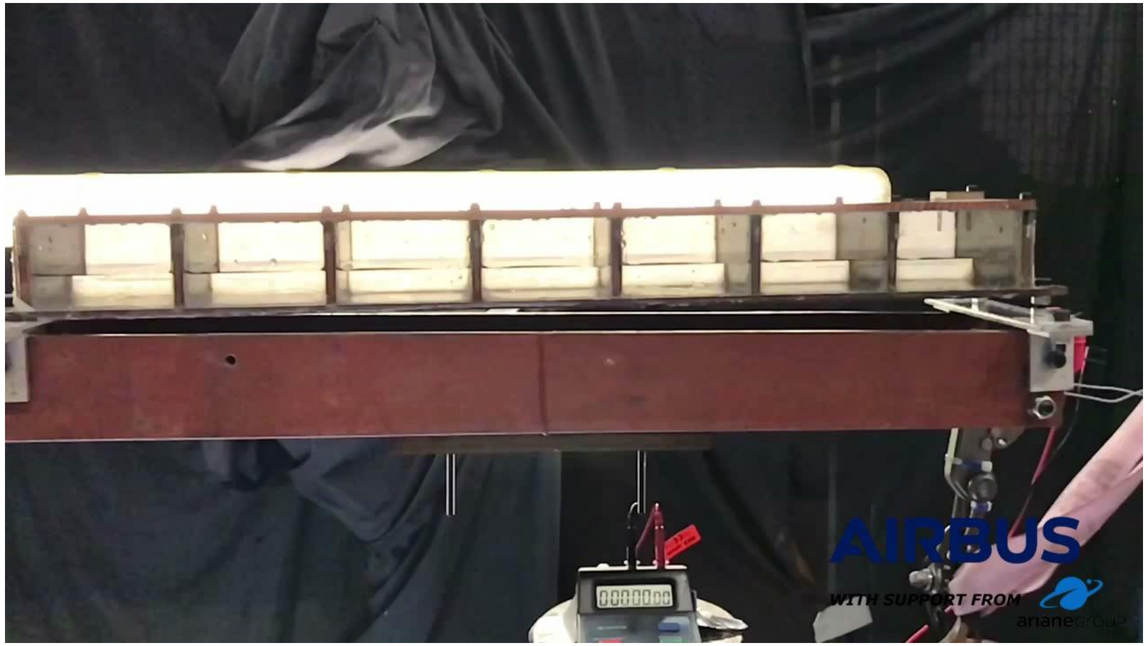
Modelling of the tanks with **their own shapes**

Nonlinear ROM for sloshing

Vertical Sloshing

Physics Overview

- It presents **Rayleigh-Taylor instabilities** that triggers at certain values of *vertical acceleration*;
- Turbolence, impacts and recomposition of the free surface provide non-conservative forces (**dissipative behaviour**);
- In ***harmonic motion*** dissipation depends on **frequency** and excitation **amplitude** (work provided to maintain the harmonic motion is equal to the work done by dissipative force)



Dissipation Metric for harmonic motion

Loss Factor/energy concept

Dissipated Work	Maximum Stored Energy
$W_D = \oint f_D dx$	U_{max}

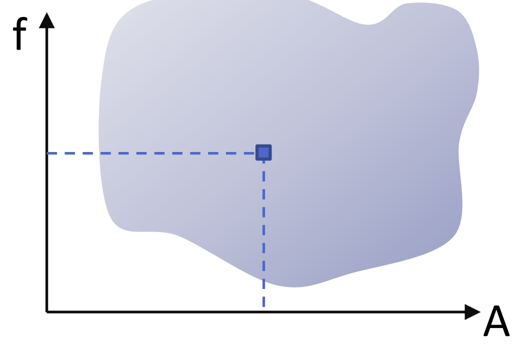
harmonic motion

Loss Factor $\eta := \frac{W_D}{2\pi U_{max}}$

Linear Systems
System characteristic related to the dissipation capabilities

Nonlinear Systems
Quantity dependent on the *imposed harmonic motion*

Loss Factor can be computed for each **frequency (f) / amplitude(A)** pair



Loss Factor Map

or

Energy Map

- It allows us to define a **dissipation metric** for the ***fluid (+ tank geometry) system*** in **harmonic motion** (system characterization of the vertical sloshing **dissipative behavior**)

Data Providers

To study vertical sloshing dissipative behavior, a data set containing the **dissipative forces** should be available!

Potential Data Providers are:

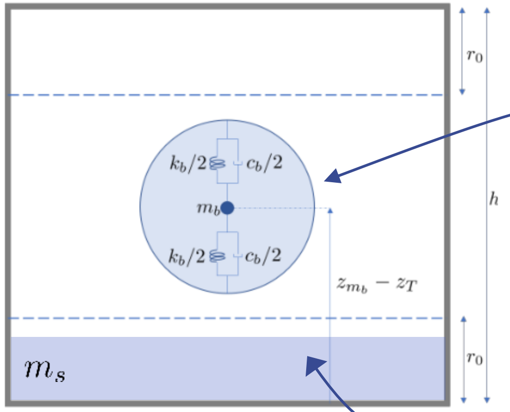
- **Experiments**: Cell Load Sensors
- **Numerical simulations**: CFD capable to describe dissipative fluid mechanism

In **steady harmonic motion**

Not available at the moment!

A **NL ROM** has been developed ***but*** obtained by available **decaying-harmonic experimental data** (by Universidad Politecnica de Madrid, UPM)

Bouncing Ball Model

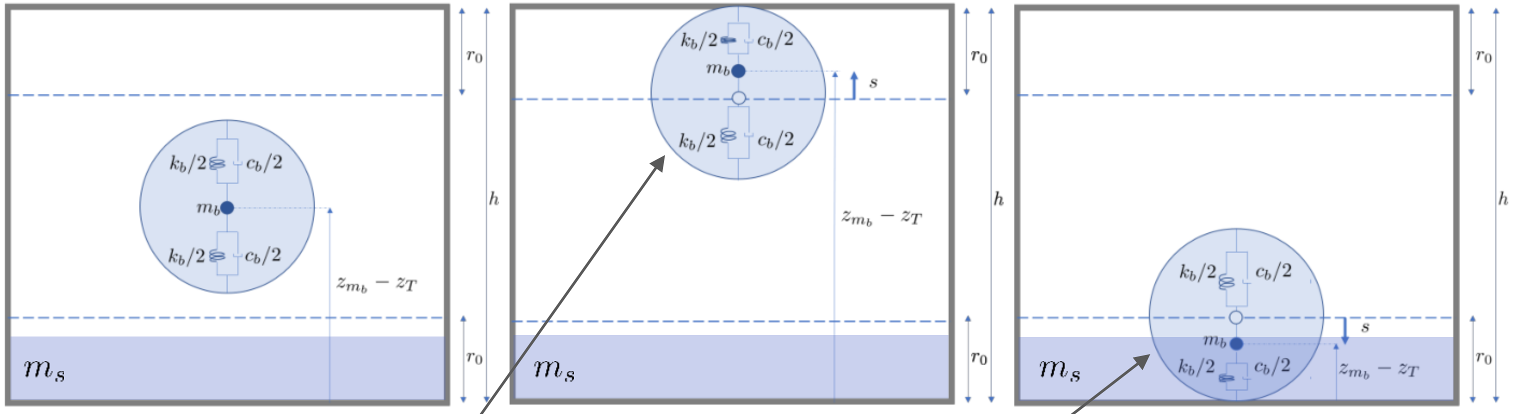


Vertically bouncing ball able to *emulate the vertical sloshing impacts mechanism and dissipation*

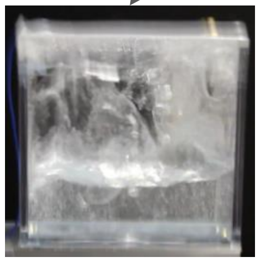
Frozen fluid mass

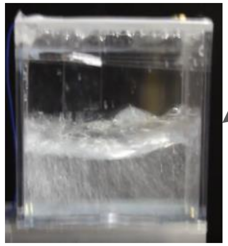
A Bouncing Ball NL ROM

- 1DoF mono-dimensional system in which **sloshing forces** are replaced by the forces exchanged between the tank wall and a ball bouncing inside the rigid tank.



(a) Floating in the air (b) Impacting on the ceiling (c) Impacting on the floor





Equation of motion of the bouncing ball

$$m_b \ddot{z}_{m_b} = -m_b g + F_b(z_{m_b}(t), z_T(t))$$

Displacement of m_b with respect to the geometric center of the ball.

Viscoelastic Forces

$$F_b(s(t), \dot{s}(t)) = k_b s + c_b \dot{s}$$

$$\begin{cases} k_b = \hat{k}_b f_{nl}(s) \\ c_b = \hat{c}_b f_{nl}(s) \end{cases}$$

$$f_{nl}(s) = \left(1 + \frac{\alpha s^2}{r_0 - |s|}\right)$$

Penalty Function

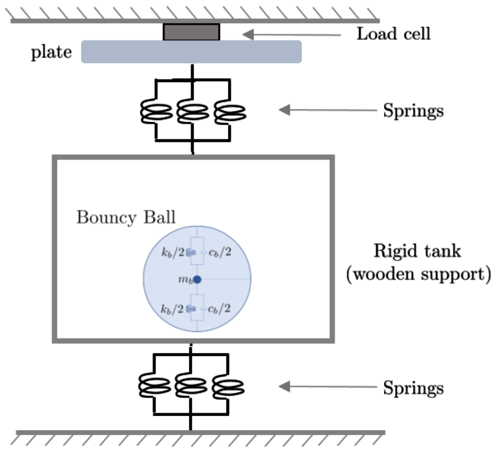
Frozen Mass $m_s = \beta m_f$

Sloshing Fluid Mass $m_b = (1 - \beta) m_f$

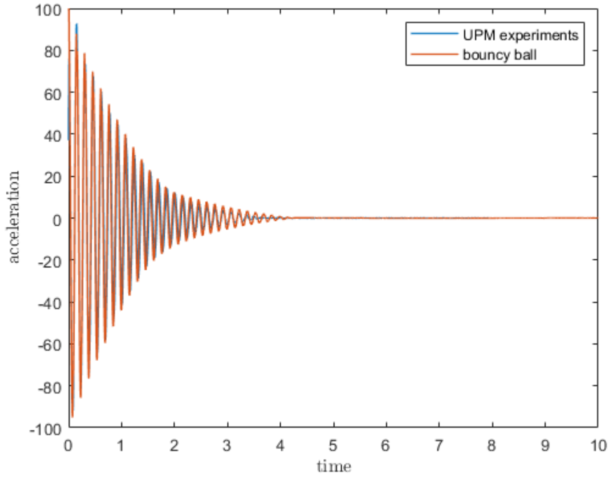
Design variables

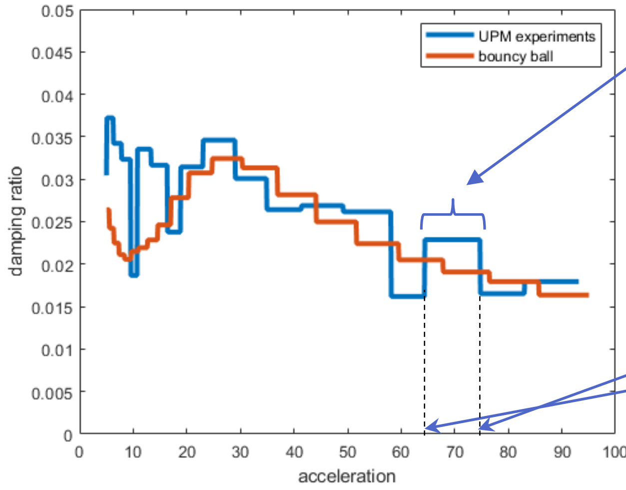
Bouncing ball parameters by experimental data

- Having the experimental data of the ***free response*** analysis performed by ***UPM*** available, it was decided to find the main **parameters of the bouncing ball** by means of an **optimization process**



Virtual Experimental Setup
UPM Experimental Setup

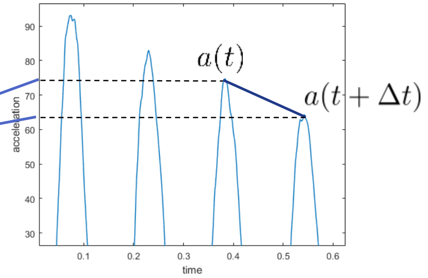




Damping ratio assumed **constant** between two consecutive peaks of the acceleration amplitude

$$\zeta = \frac{\log(a(t+\Delta t)) - \log(a(t))}{\omega_n \Delta t} = \eta/2$$

$$\omega = \omega_n$$



- The **instantaneous damping ratio** can be obtained from the **logarithmic decay** as a function of the **acceleration amplitude**

- By exploiting a **Simulink© model** (including the bouncing ball ROM), the **optimal parameters** are obtained **minimizing the distance** between the damping ratio curves (blue and red)

Optimal bouncing ball parameters

h	0.06 m
m_f	0.18 kg
r_0	0.0212 m
$\hat{k}_b$	1000 $\mathrm{Nm^{-1}}$
$\hat{c}_b$	8.21 $\mathrm{Nsm^{-1}}$
α	250 $\mathrm{m^{-1}}$
β	0.17

Design variables (r_0, $\hat{k}_b$, $\hat{c}_b$, α, β)

Metrics for steady/decaying harmonic motion

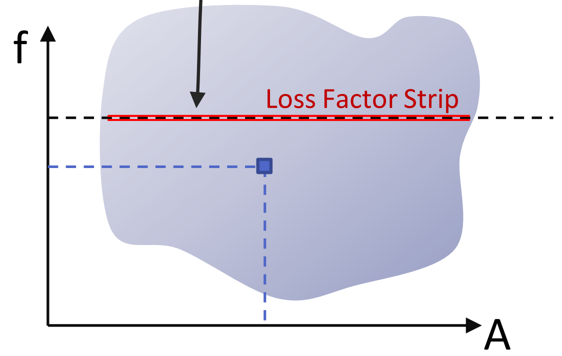
Loss Factor Map "Strip"

Dissipation metric for a **Deacaying-Harmonic Motion (DHM)**

By means of a *fitting/optimization process* the ***bouncing ball ROM*** is obtained

After, a **Steady Harmonic Motion (SHM)** may be performed for building a **Loss-factor / energy map**

The validity of the bouncing ball identification is guaranteed if **similar dissipation curves** are obtained at the selected strip



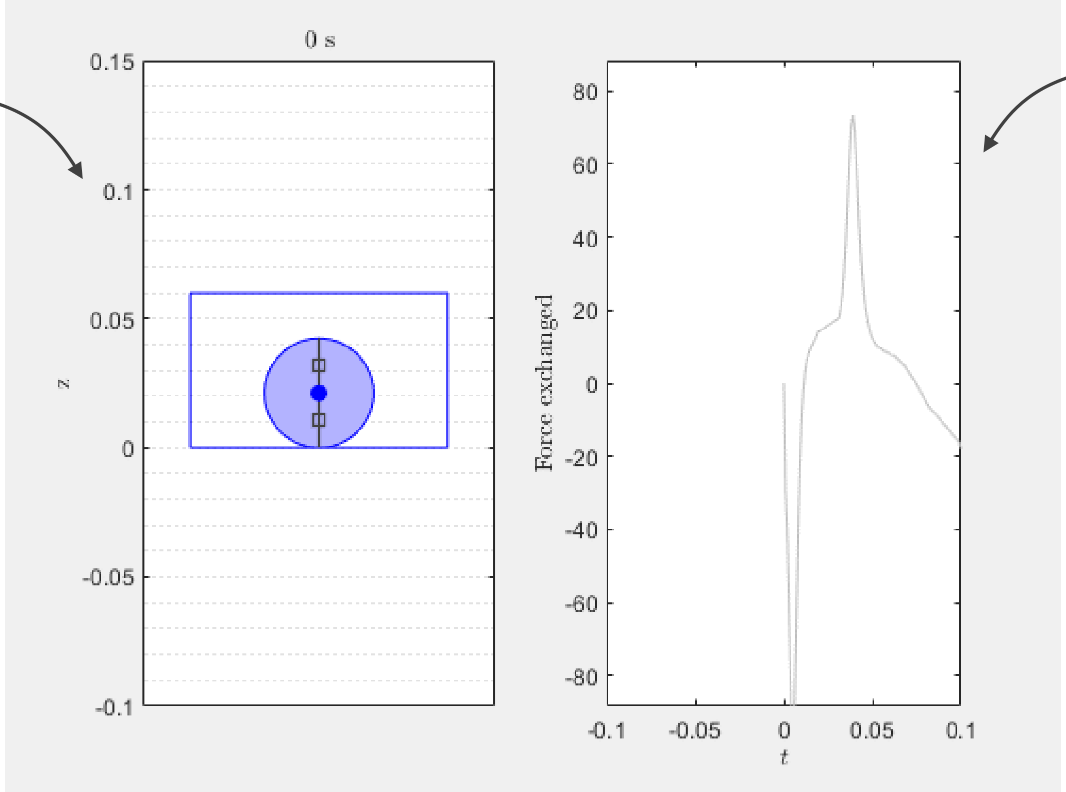
Natural frequency of the UPM structural configuration

Loss Factor / energy Map

Tank steady-harmonic-motion simulation

- The system (***isolated*** tank with bouncing ball inside it) is excited via an **imposed harmonic motion of the wall**, having excitation amplitude and frequency respectively equal to **0.07m** and **43.98 rad/s (7 Hz)**

Resulting **response** of the bouncing ball

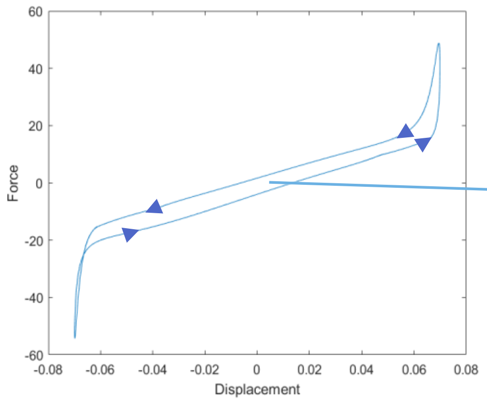


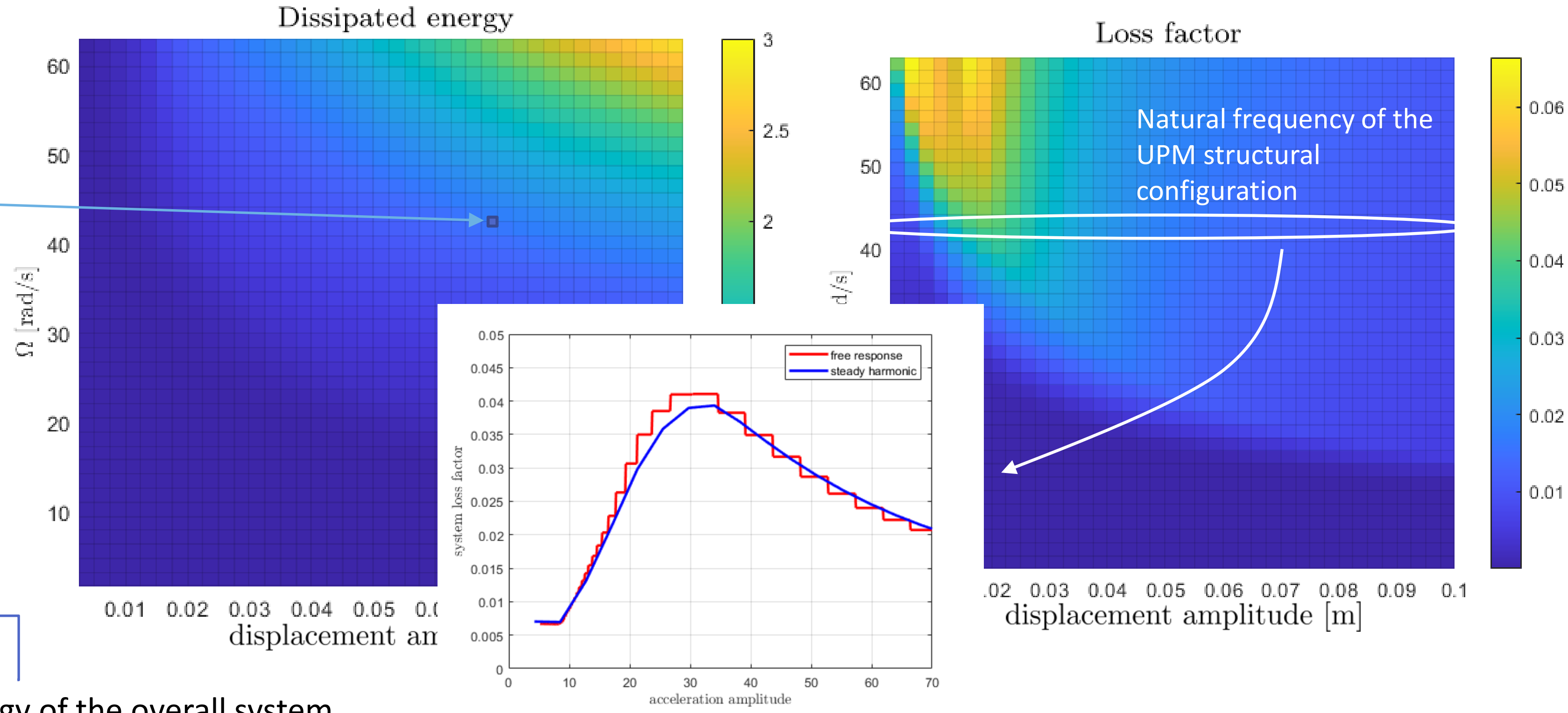
Viscoelastic forces generated by the **impacts**

Go back to loss-factor/energy Map

- It is possible to obtain an estimation of the **dissipation** by means of the **hysteresis cycle** (force v.s. displ.)

- The analysis is then carried out for different values of **excitation amplitude** and **frequencies** by evaluating the **dissipated energy** and the **loss factor**:





System Loss Factor

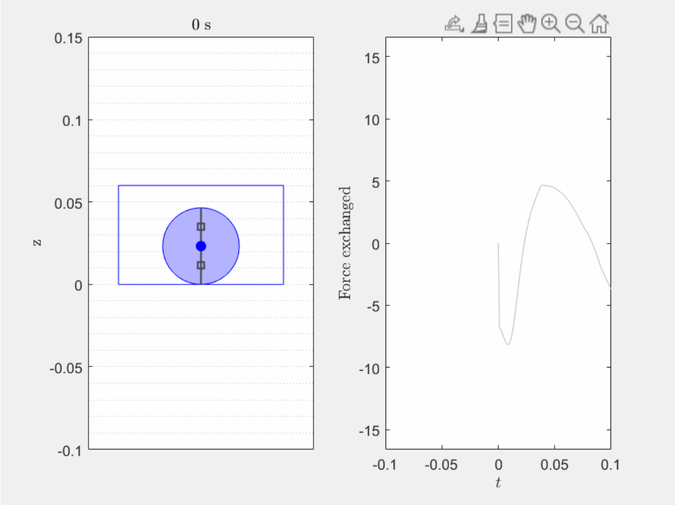
$$\eta := \frac{W_D}{\pi(m_f+m_s)\Omega^2 A^2}$$

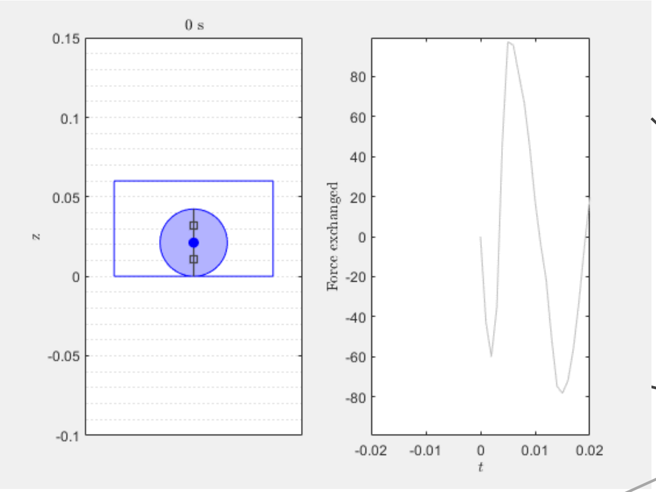
Kynetic energy of the overall system

NL ROM capabilities

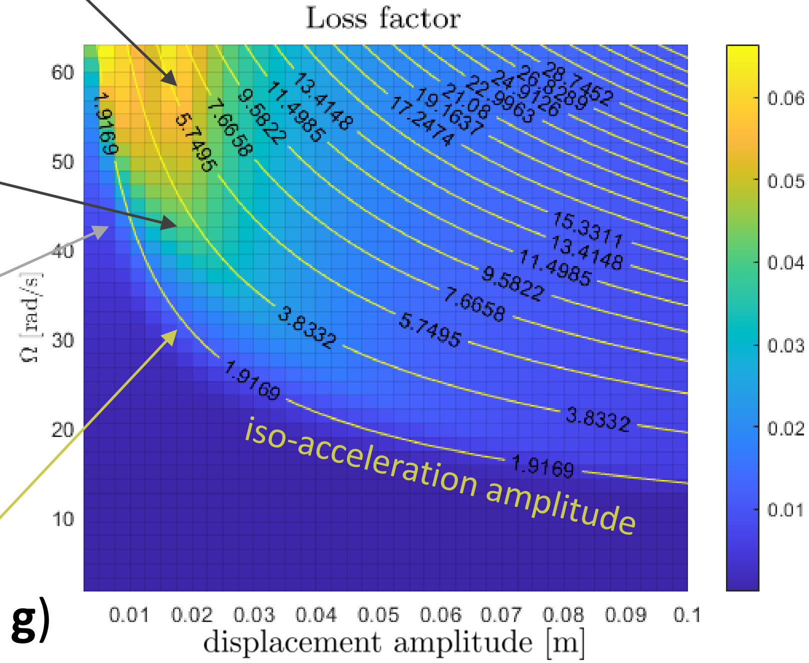
Any Rayleigh-Taylor instabilities?

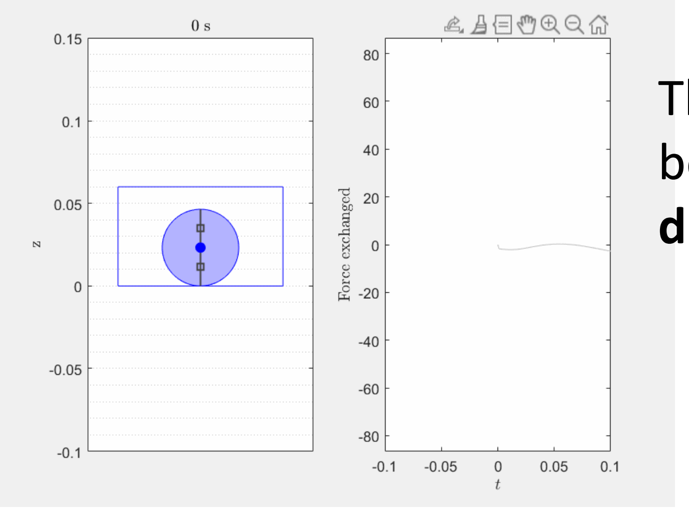
Beyond the frontier **there are impacts** that dissipate energy





Region with **maximum values** of the loss factor





The lack of impacts before the border leads to **no energy dissipation**

This Iso-acceleration curve (about **2 g**) looks like a **Rayleigh-Taylor instability** frontier

Another NL ROM version of the system
Neural Network based ROM

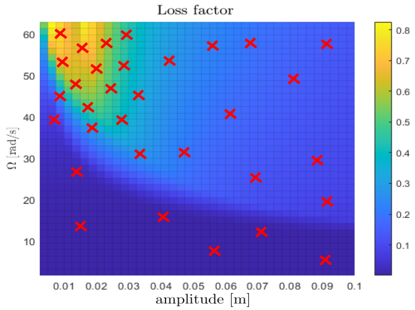
Identification of the Sloshing Model

- **Bouncing Ball Model** considered as a replacement of Sloshing to generate data (*training*)
- **Input/Output Neural Network** used for System Identification

Training Phase

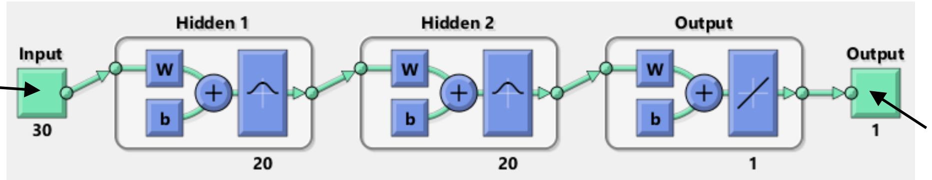
Different types of system identification signals:
- Stochastic Inputs
- Steady Harmonic Inputs
- Random Narrow Band Inputs

Large **DoE** with frequency-amplitude pairs in the **energy map** domain



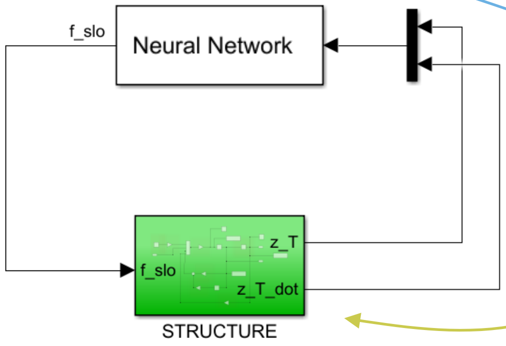
Feedforward Neural Network

Vertical imposed motion and velocity



Sloshing force

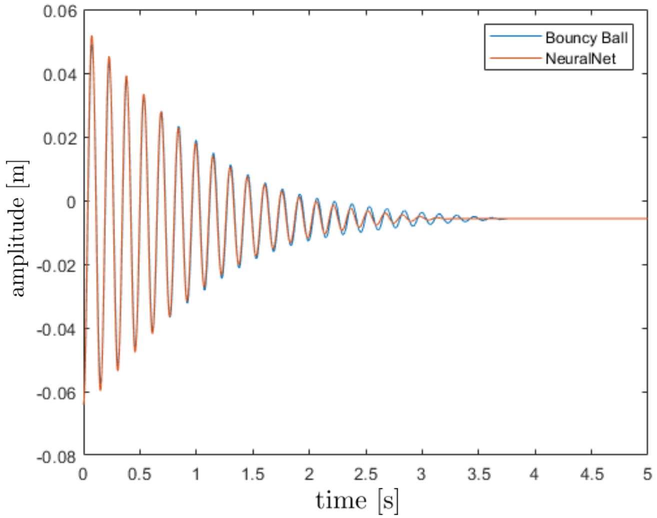
Neural Network in the closed-loop Simulink© framework



Bouncing Ball model

UPM structural configuration

Closed-Loop Free Response simulating UPM experiment



Concluding Remarks

- **Linear ROM** for sloshing fluid into aircraft tank has been developed, showing a good level of *coupling description* for flexible aircraft aeroelastic formulation.
  PROS: The ROM capabilities allows to include the sloshing effects in a not-standard way for global stability analysis (flutter).
  CONS: Important *dissipative effects cannot be described* by linear ROM.

- **Nonlinear ROM** are identified as *energy map* giving the fluid dissipated energy as function of amplitude and frequency in a **steady harmonic motion.**

  - ✓ For **decaying harmonic motion** (data provided by experiments) the above concept was adapted to identify a nonlinear ROM consisting of a ***bouncing ball*** into a closed box. The obtained nonlinear ROM was also capable to provide a synthetic *energy map* exhibiting the onset of a **RT instability**.
  - ✓ By using the bouncing ball data, a ***neural network*** *has been* identified for providing another nonlinear ROM version for the same system.

Thank you!

The research leading to these results has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 815044.