Sloshing induced damping in vertically vibrating systems

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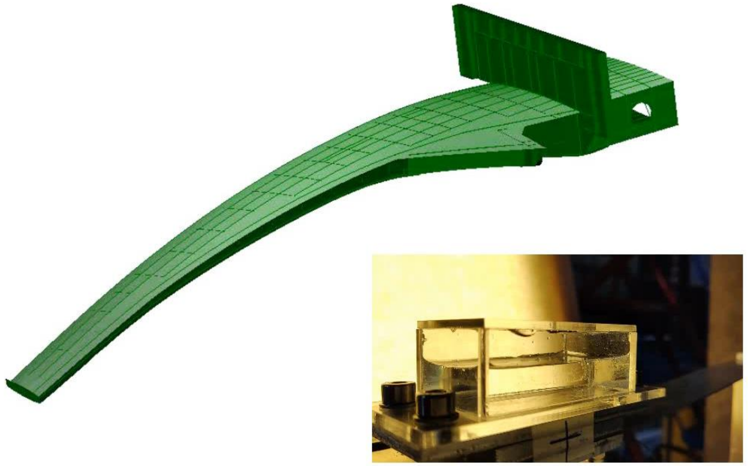
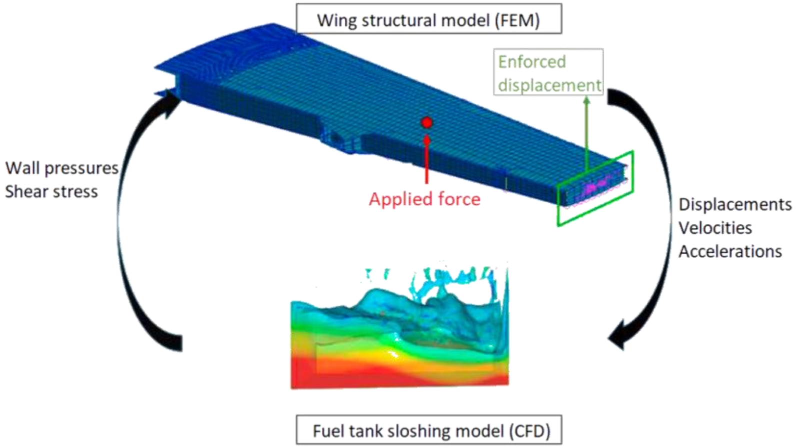
Overview

- Introduction – fuel sloshing for loads alleviation
- Experimental investigation of liquid sloshing in a SDOF system
- Sloshing surface identification
- Numerical modelling – smoothed particle hydrodynamics
- Conclusions

Introduction

- All aircraft are subjected to loads throughout ground and flight operations, defining the structural sizing and weight.
- Active and passive loads alleviation technologies reduce dynamic loads due to atmospheric gusts and turbulence, leading to more fuel-efficient aircraft designs.
- H2020 SLOW-D project considering wing fuel-sloshing for loads alleviation via increased effective damping.
- **Objective** – experimental and numerical investigation of a vertically excited single degree-of-freedom system, coupled to a liquid filled container. Comparison of SPH modelling against experimental results.

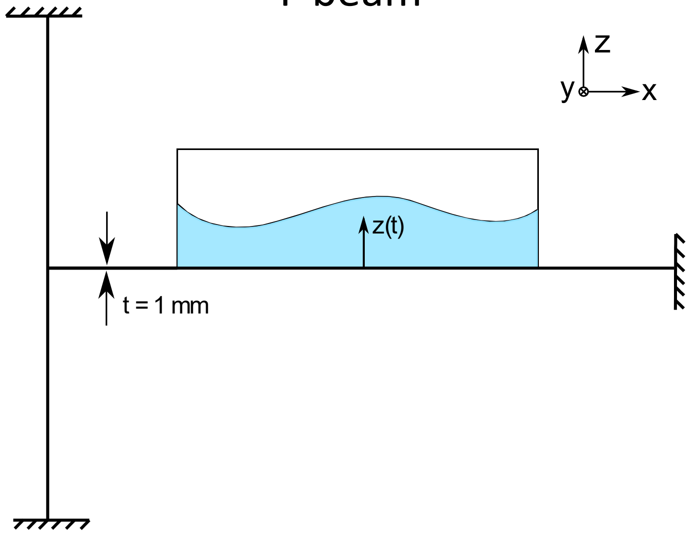




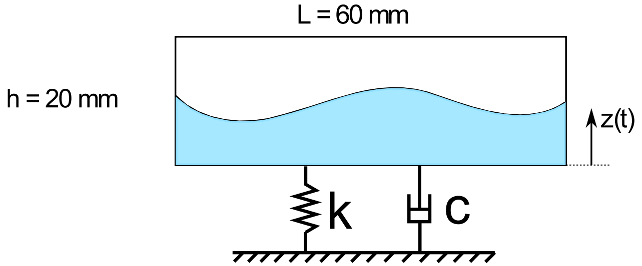
Experiment: SDOF system

- Experimental rig aims for lightly damped, linear, vertically constrained motion.

‘T-beam’



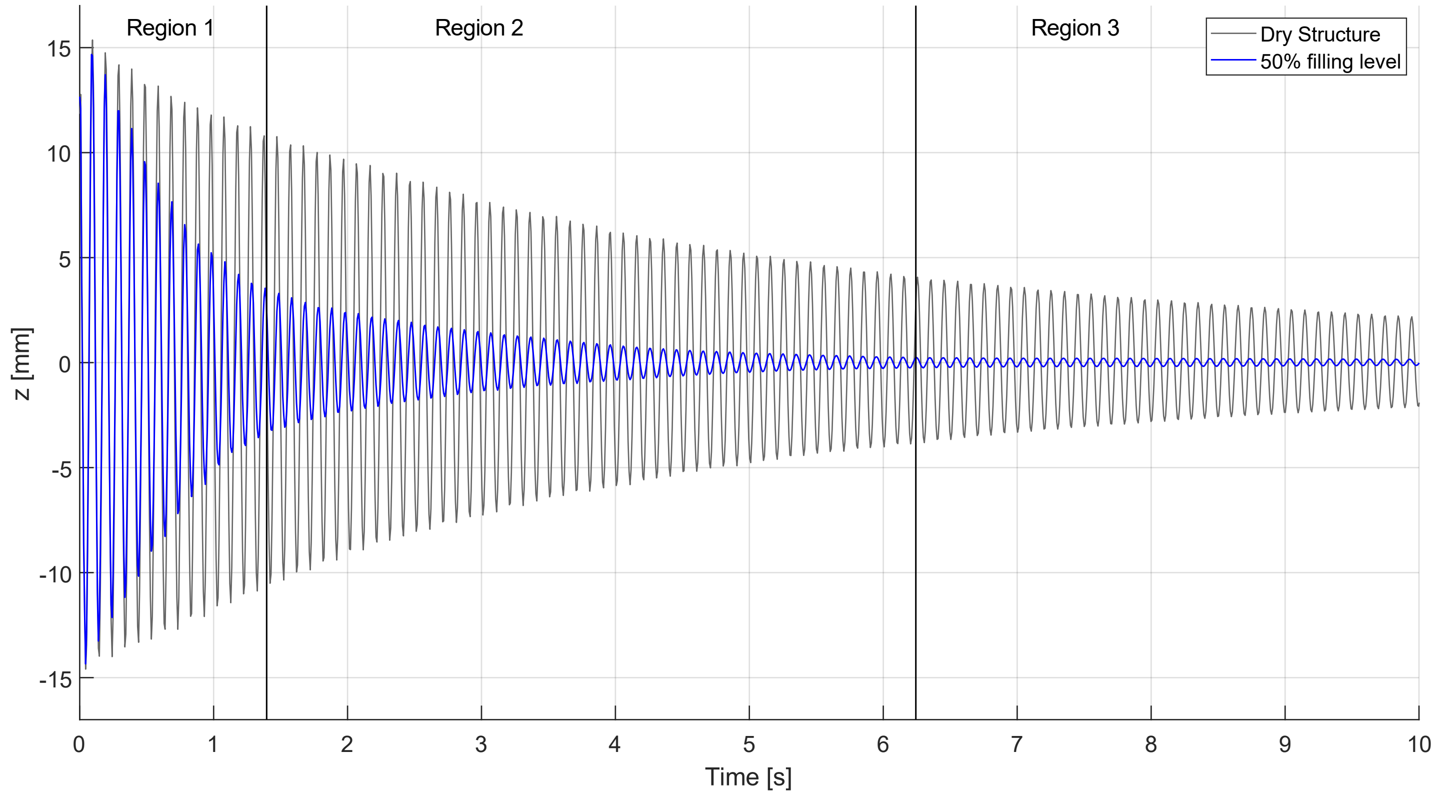
SDOF equivalent



- Two steel strips joined in a ‘T’ shape.
- Joint allows rotation and translation of horizontal beam left constraint.
- Primarily vertical motion, with linear displacement up to 14 times beam thickness.

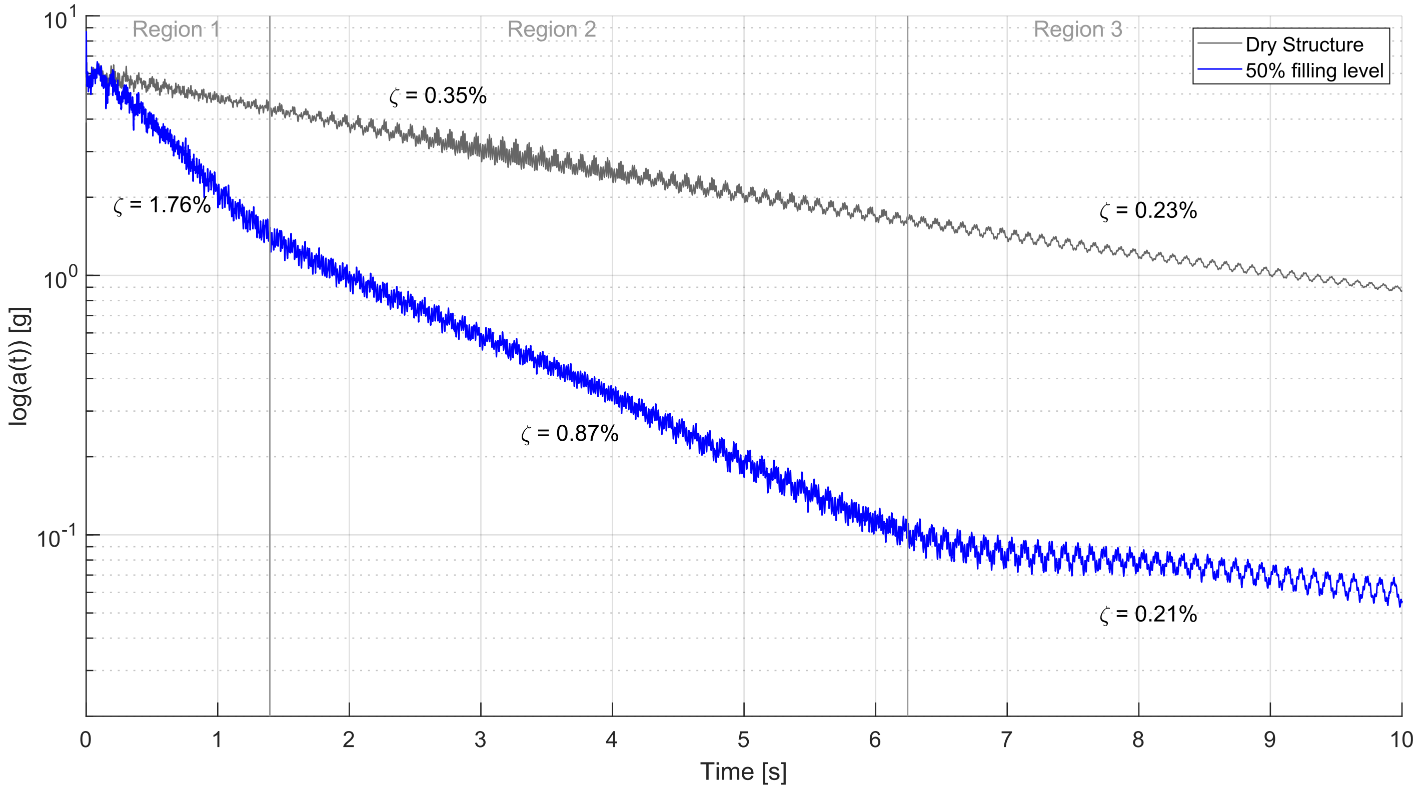
m_{eq}	0.275	kg
k	1.1	N/mm
f	10.05	Hz
ζ	$0.23 - 0.34$	$\%$

Experiment: Dynamic response



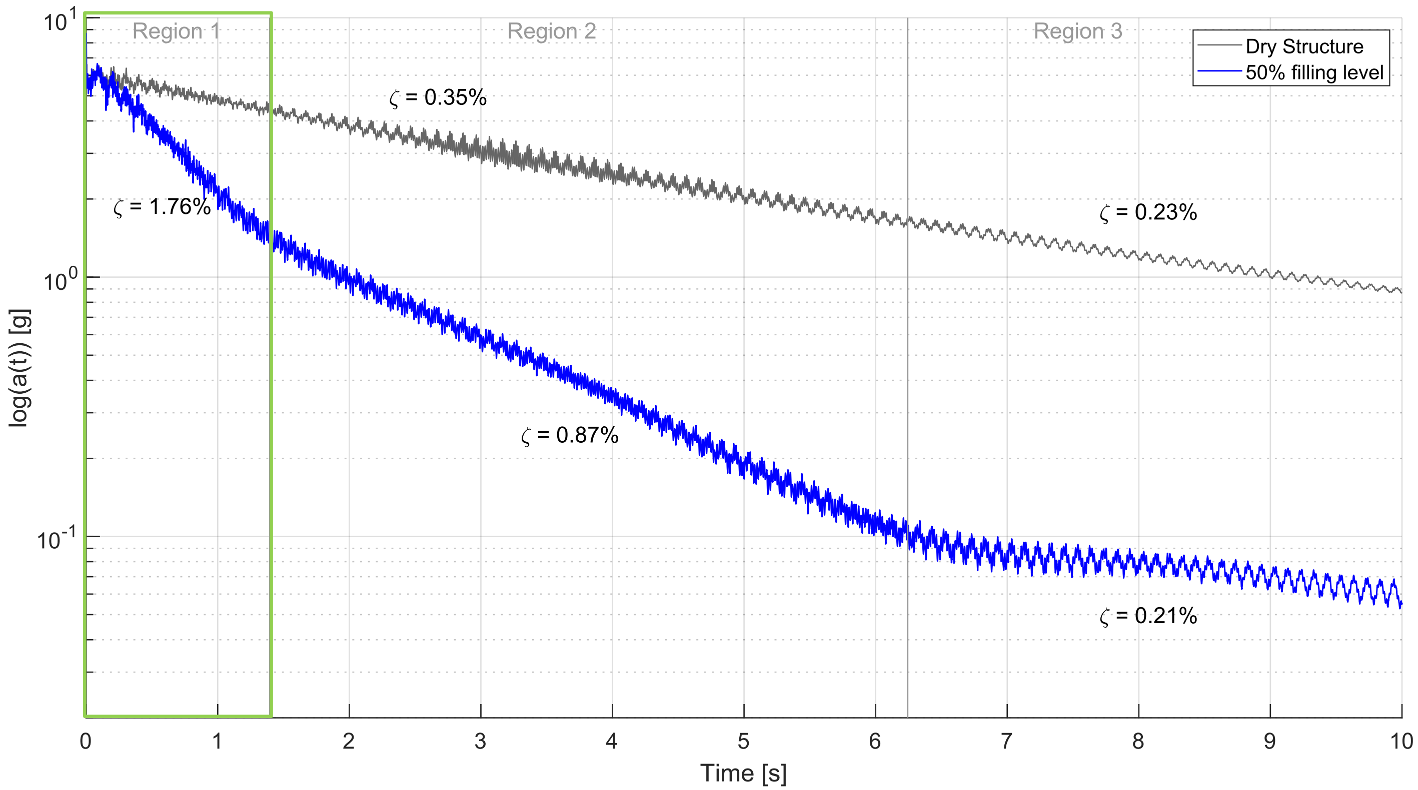
- Tank displacement during free vibration, in wet and dry conditions.
- Significantly increased dissipation with fluid sloshing.
- 50% filling level considered throughout this work, maximises fluid induced damping.
- Systems are kept dynamically comparable, using added weights to maintain constant frequency between wet and dry cases.

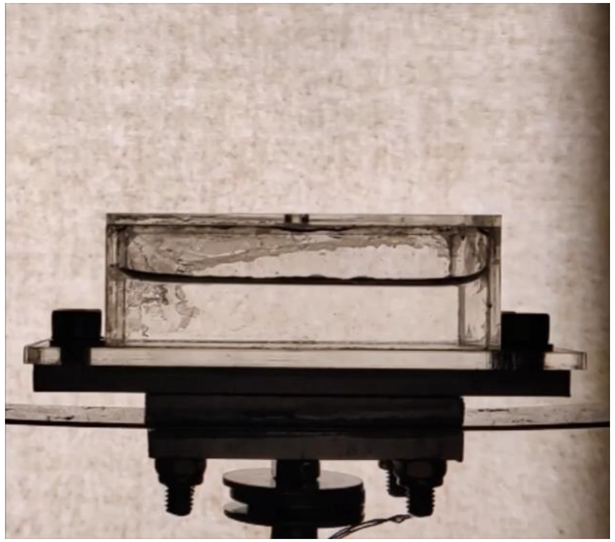
Experiment: Dynamic response



- Logarithmic envelope of acceleration signal vs time.
- Dry structure has a lightly damped, bi-linear response.
- Logarithmic plot reveals three linear dissipation regions in wet conditions. Referred to as R1-3.
- Each region has a distinct flow regime and corresponding damping ratio.

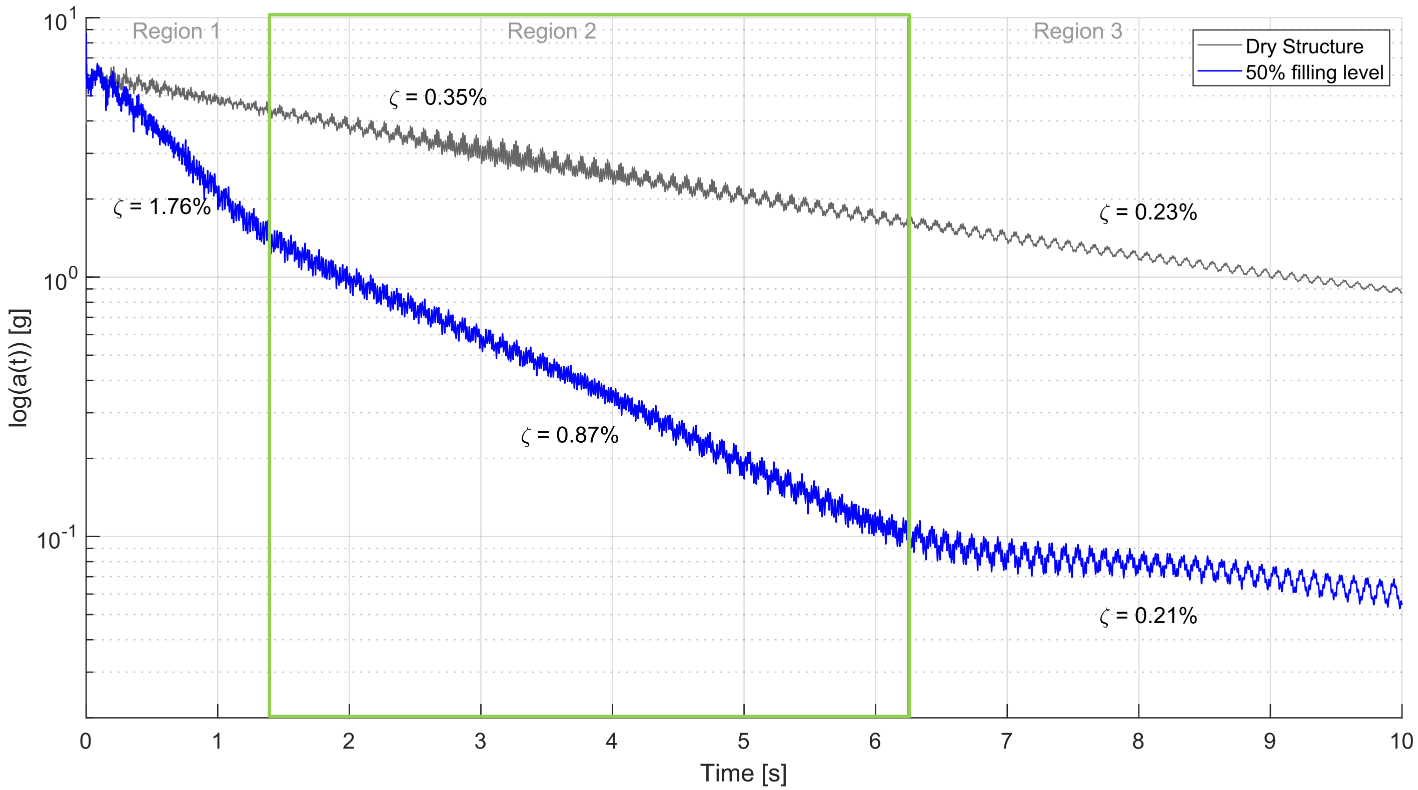
Experiment: R1

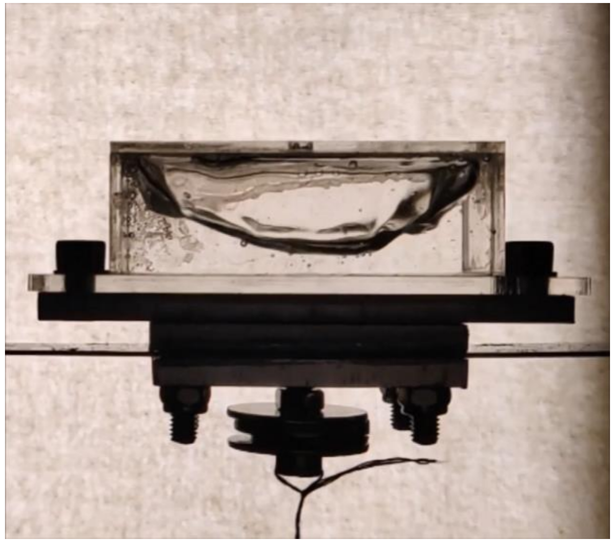




- R1 presents the largest damping ratio.
- Turbulent fluid motion with highly complex free surface behaviour.
- Strong impacting between fluid and horizontal tank surfaces.
- Presence dependent on peak accelerations exceeding 1g.

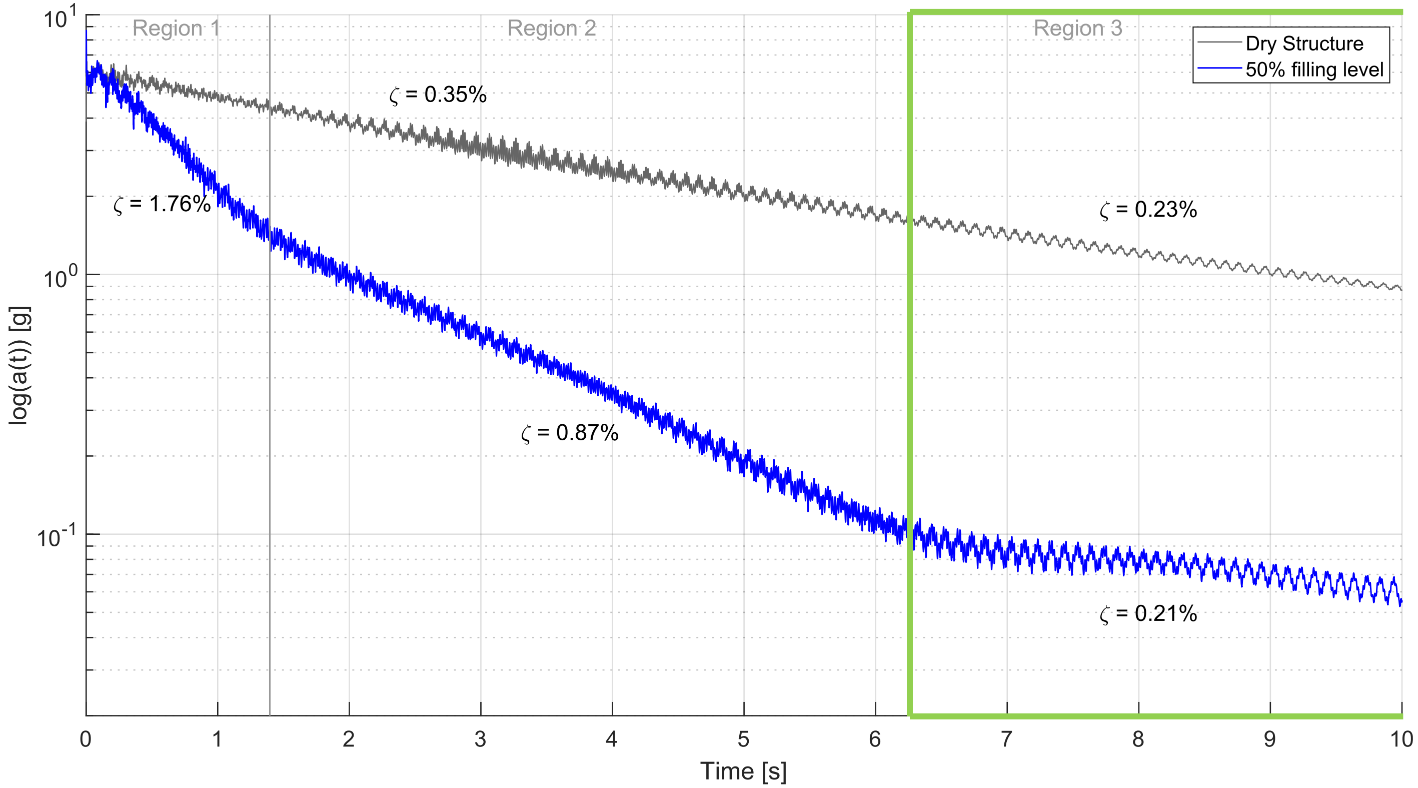
Experiment: R2

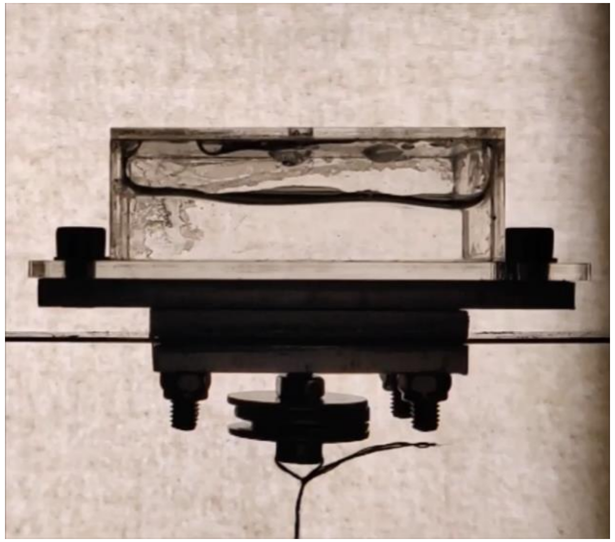




- R2 dominated by parametric excitation of fluid within the first symmetric sloshing mode.
- Free surface motion has frequency at half the excitation.
- Considerable duration, still large source of energy dissipation.
- R2 main focus of this work.

Experiment: R3

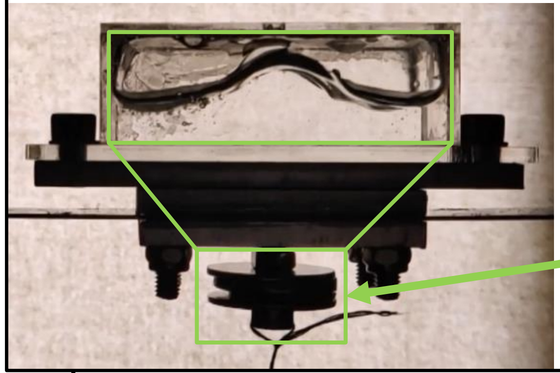




- Fluid mostly stationary within R3, with some free-surface oscillation.
- Fluid behaves as if a fixed mass, damping ratio returns to dry values.

Experiment: Surface identification

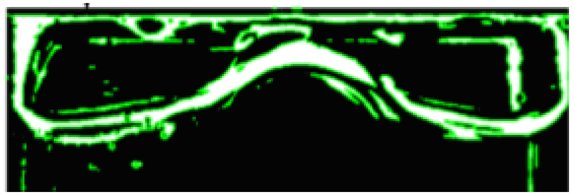
1. Tank boundaries identified



Mark image

2. Binarize frame

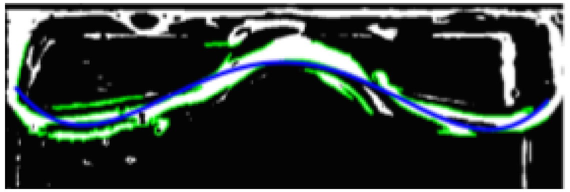
3. Identify white regions (surface boundaries)



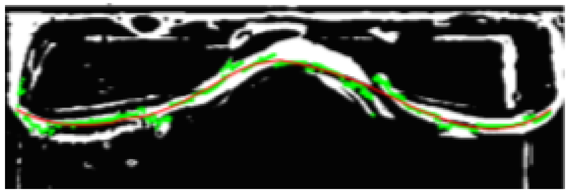
4. Cut boundaries and small point clusters

5. Keep mean vertical point at each x station

6. Polynomial fit to data and outliers removed



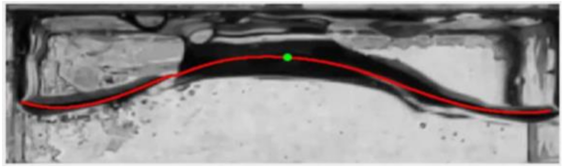
7. Data smoothed with Savitzky-Golay filter



8. Repeat 6 with tighter tolerances

9. Smooth data with S-G filter

Surface identified!



- Process extracts surface motion during R2 and R3 regimes from high-speed footage.
- Multi-step process designed to robustly identify a 2D surface while avoiding 3D influences.
- Binarized frame provides initial set of surface points.
- Multiple filters used with reducing tolerances, to carefully remove outlying points.
- Processes repeated for every frame.

SPH: Introduction

- Smoothed particle hydrodynamics chosen method for modelling free surface flows.
- Lagrangian in nature – mesh free.
- Continuum equations of motion discretised onto a set of fixed mass particles:

$$\begin{cases} \dfrac{D\rho}{Dt} = -\rho \nabla \cdot \mathbf{u} \\ \dfrac{D\mathbf{u}}{Dt} = -\dfrac{1}{\rho}\nabla P + \dfrac{\nu}{\rho}\nabla^2 \mathbf{u} + \mathbf{g} \\ P = F(\rho) \end{cases}$$

- Fundamental interpolation reconstructs continuous field via a spatially smoothed summation over a neighbourhood of particles:

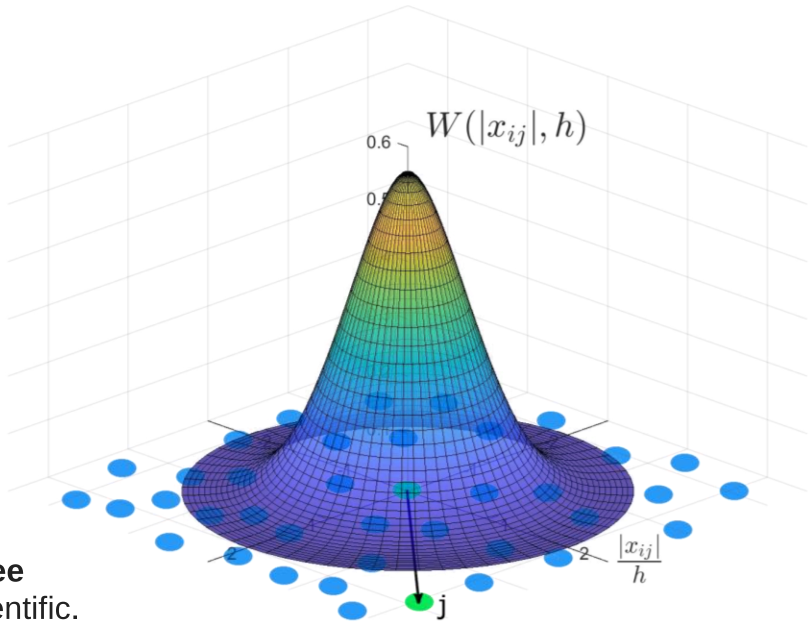
$$f(\mathbf{x}_i) \approx \sum_j f(\mathbf{x}_j) W(\mathbf{x}_i - \mathbf{x}_j, h) \frac{m_j}{\rho_j}$$

- Classically used to evaluate particle density:

$$\rho(\mathbf{x}_i) \approx \sum_j m_j W(\mathbf{x}_i - \mathbf{x}_j, h)$$

- Spatial gradients evaluated similarly:

$$\nabla f(\mathbf{x}_i) \approx -\sum_j f(\mathbf{x}_j) \nabla W(\mathbf{x}_i - \mathbf{x}_j, h) \frac{m_j}{\rho_j}$$



Liu, G.R. and Liu, M.B., 2003. **Smoothed particle hydrodynamics: a meshfree particle method**. World scientific.

SPH: Coupled Formulation

- Single phase WCSPH formulation:

$$\frac{D\mathbf{u}_i}{Dt} = -\sum_j m_j \left[\frac{P_j}{\rho_j^2} + \frac{P_i}{\rho_i^2} + \Pi_{ij}\right] \nabla W_{ij}$$
$$+ \sum_j m_j \nu \frac{\rho_i + \rho_j}{\rho_i \rho_j} \frac{\mathbf{x}_{ij} \cdot \nabla W_{ij}}{|\mathbf{x}_{ij}|^2 + 0.001h^2} \mathbf{u}_{ij}$$
$$+ \mathbf{a}_{ST,i} + \mathbf{g}$$

$$\frac{D\rho_i}{Dt} = \sum_j m_j \mathbf{u}_{ij} \cdot \nabla W_{ij} + \delta h c_0 \mathcal{D}_i$$

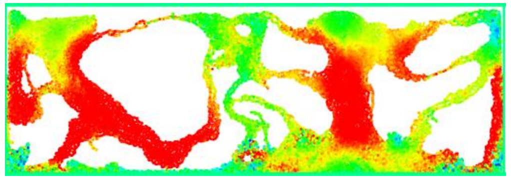
$$P = \frac{\rho_0 c_0^2}{\gamma} \left[\left(\frac{\rho}{\rho_0}\right)^{\gamma} - 1\right]$$

- Temporal integration with Newmark beta
- Wendland C2 kernel

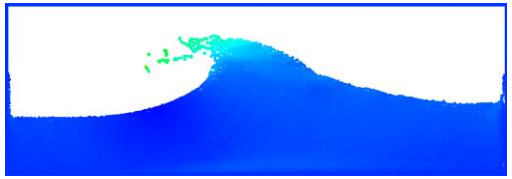
- SPH-structure coupled model:
  - Partitioned approach, fluid and structural domains solved separately.
  - Strong (or tight) coupling; ensures exact temporal synchronisation and conservation of energy in FSI system.
  - Simple 1DOF mass spring damper model used:

$$M\ddot{x} + C_1\dot{x} + C_2|\dot{x}|\dot{x} + Kx = F_{sph}$$

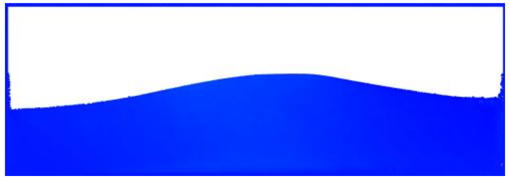
  - Dry structure damping parameters determined through optimisation procedure.



R1



R2

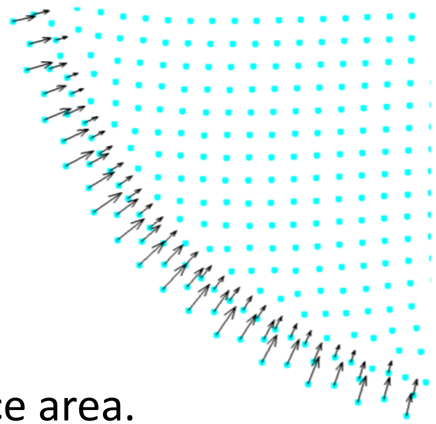


R3

SPH: Surface tension

- Surface tension effects potentially non-negligible at small experimental tank sizes (60 mm width).

- Two main methodologies for modelling surface tension in SPH:



- **Pairwise force**
- Based upon cause, cohesive forces between molecules.
- Tartakovsky and Meakin 2005:

$$\mathbf{F}_{ij} = s_{ij} \cos\left(\frac{1.5\pi}{3h}|\mathbf{x}_{ij}|\right) \frac{\mathbf{x}_{ij}}{|\mathbf{x}_{ij}|}, \quad |\mathbf{x}_{ij}| \leq h$$

- Simple implementation with intrinsic wetting.
- Extensive calibration required.
- Internal forces in the presence of particle disorder – induced viscosity like effects.

- **Continuum surface force**
- Based upon effect, minimising surface area.
- Morris 2000:

$$(\mathbf{a}_{ST})_i = -\frac{\sigma}{\rho_i}(\nabla \cdot \hat{\mathbf{n}})_i \mathbf{n}_i$$

- Prescence limited to free surface
- Accurate computation of surface normals required.
- Wetting behaviour not completely defined.
- Thin features can't be entirely resolved with a single phase.

SPH: Surface tension

- Focus on robust calculation of surface normals for well behaved surface tension

- Normals corrected with ‘renormalisation matrix’:

$$\mathbf{n}_i = -\mathbf{L}_i \sum_j \nabla W_{ij} V_j$$

- Smoothed with Shepard interpolant to avoid any spurious normals:

$$\widetilde{\mathbf{n}}_i = \frac{\sum_j \mathbf{n}_i W_{ij} V_j}{\sum_j W_{ij} V_j}$$

- Normals near solid boundary prescribed according to a defined contact angle (θ):

$$\mathbf{n}_i^{mod} = f(\widetilde{\mathbf{n}}_i, \theta, y^+)$$

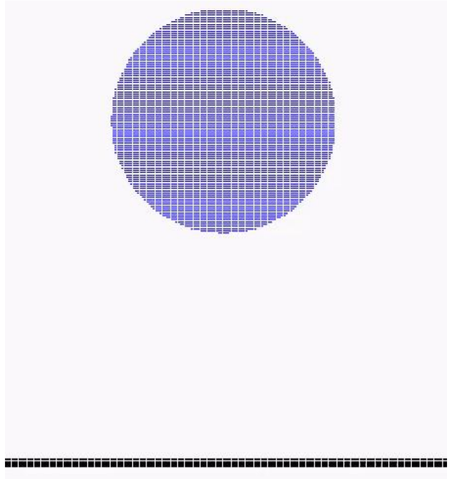
Breinlinger, T., Polfer, P., Hashibon, A. and Kraft, T., 2013. **Surface tension and wetting effects with smoothed particle hydrodynamics**. *Journal of Computational Physics*

- Curvature calculated from surface points (identified from $\mathbf{L}_i$):

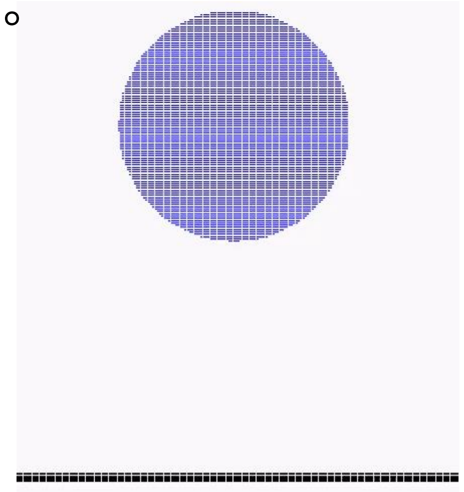
$$\kappa_i = \sum_j V_j \left(\hat{\mathbf{n}}_j^{mod} - \hat{\mathbf{n}}_i^{mod}\right) \cdot \nabla W_{ij} \qquad \kappa_i^* = \kappa_i / \mathcal{L}_i \qquad \mathcal{L}_i = \sum_j V_j W_{ij}$$

- Resulting acceleration calculated classically: $\mathbf{a}_{ST,i} = \frac{\sigma}{\rho_i} \kappa_i^* \widetilde{\mathbf{n}}_i$

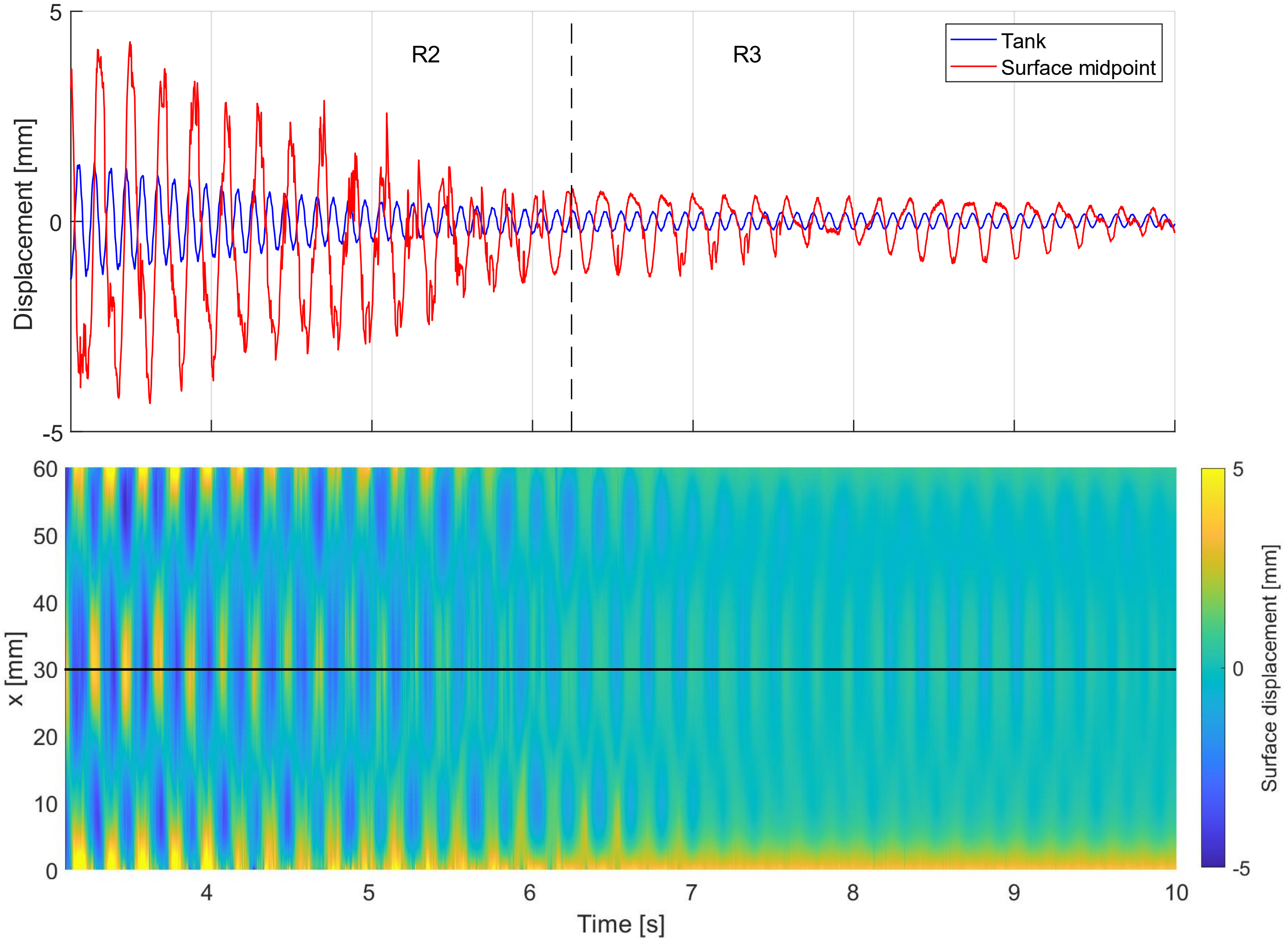
$\theta = 70°$



$\theta = 150°$



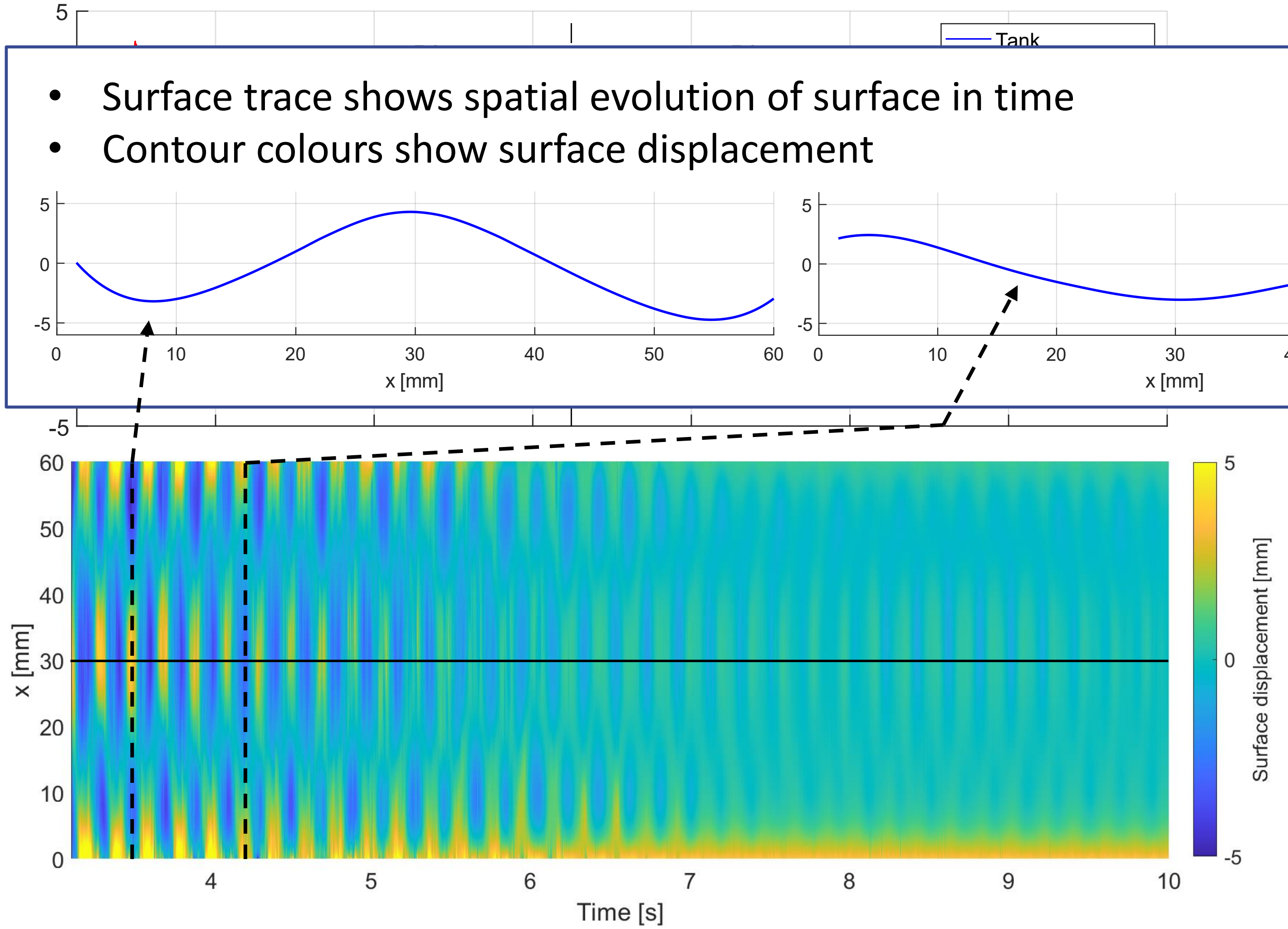
Experiment: Surface response



- Surface trace shows motion is entirely symmetric aside from small features at boundaries.
- Free surface amplitude significantly drops off into R3, along with fluid-induced damping.
- $\zeta_2 = 0.87\% \Longrightarrow \zeta_3 = 0.21\%$
- Free surface frequency (5.18 Hz) half structural frequency (10.05 Hz), typical for parametric excitation.
- Energy dissipated sustaining fluid in first symmetric sloshing mode.

Experiment: Surface response

- Surface trace shows spatial evolution of surface in time
- Contour colours show surface displacement



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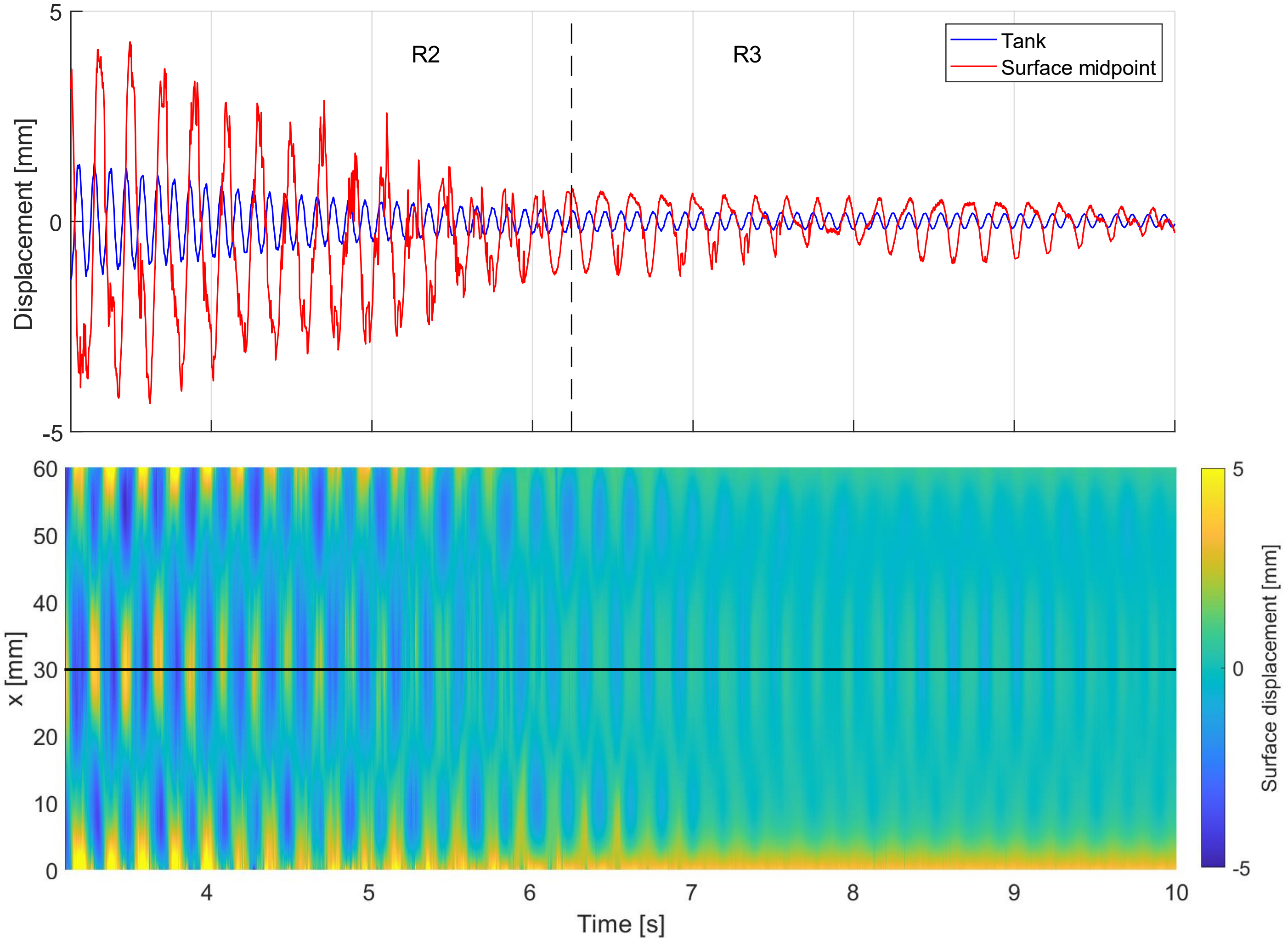

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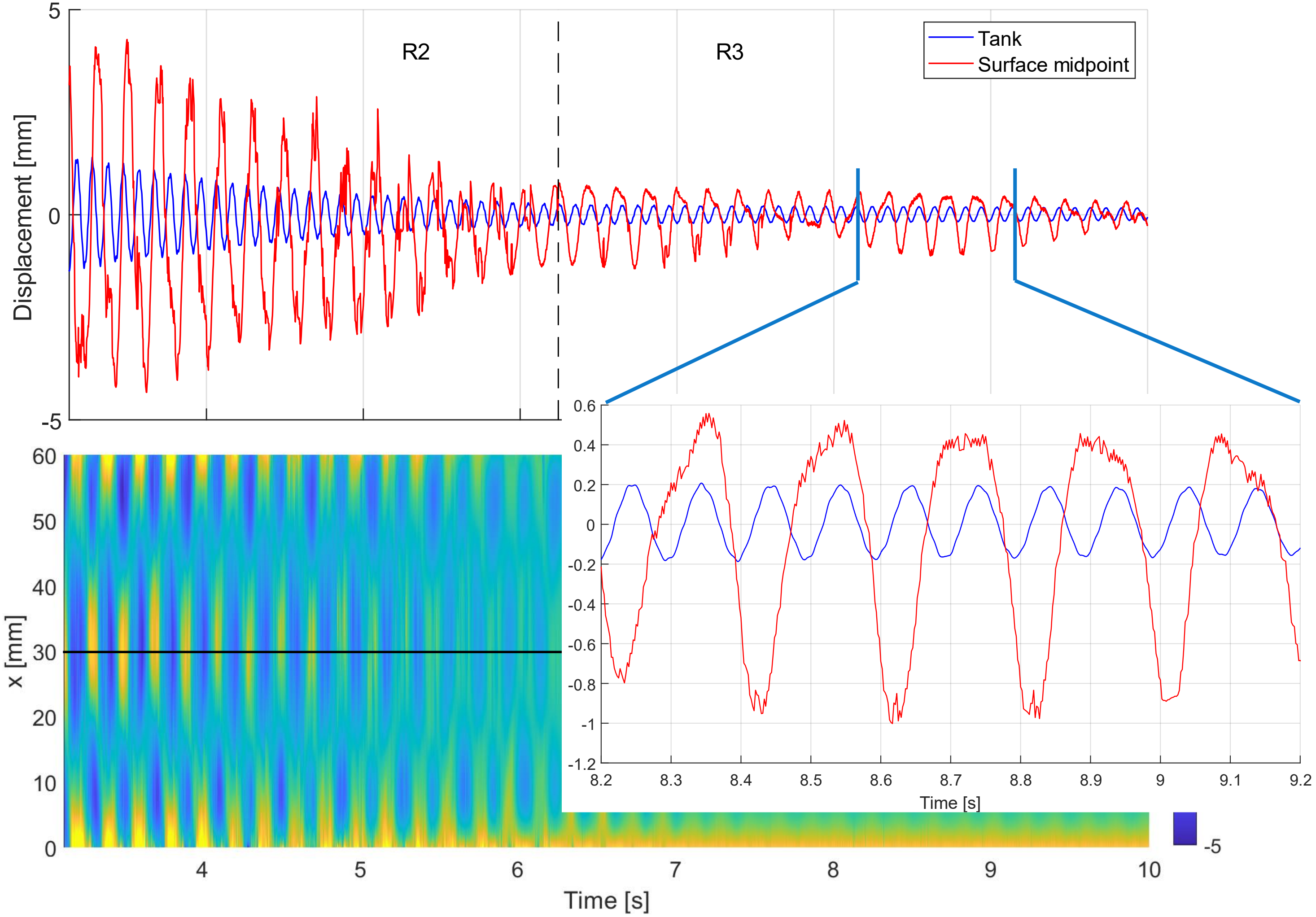

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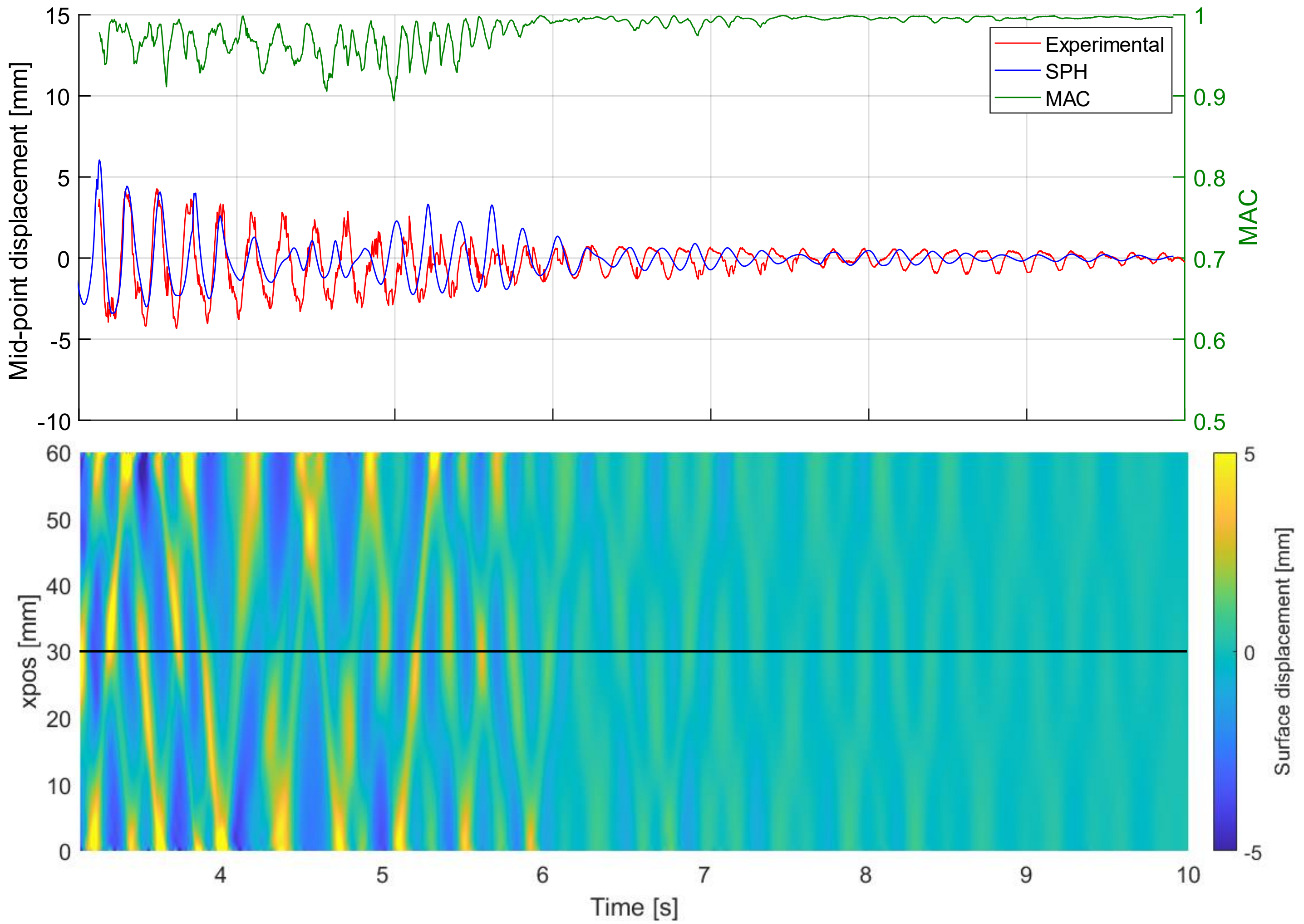
SPH: Results without surface tension



- MAC (modal assurance criterion) used as a measure of surface similarity
- Streaks across figure show strong presence of longitudinal waves moving across tank, at low frequency. First asymmetric mode at one quarter excitation frequency.
- Lack of symmetric mode under parametric excitation results in reduced R2 damping:

$$\zeta_2 = 0.54\%$$

- Slight asymmetries from initial conditions developed through R1, not attenuated during R2.

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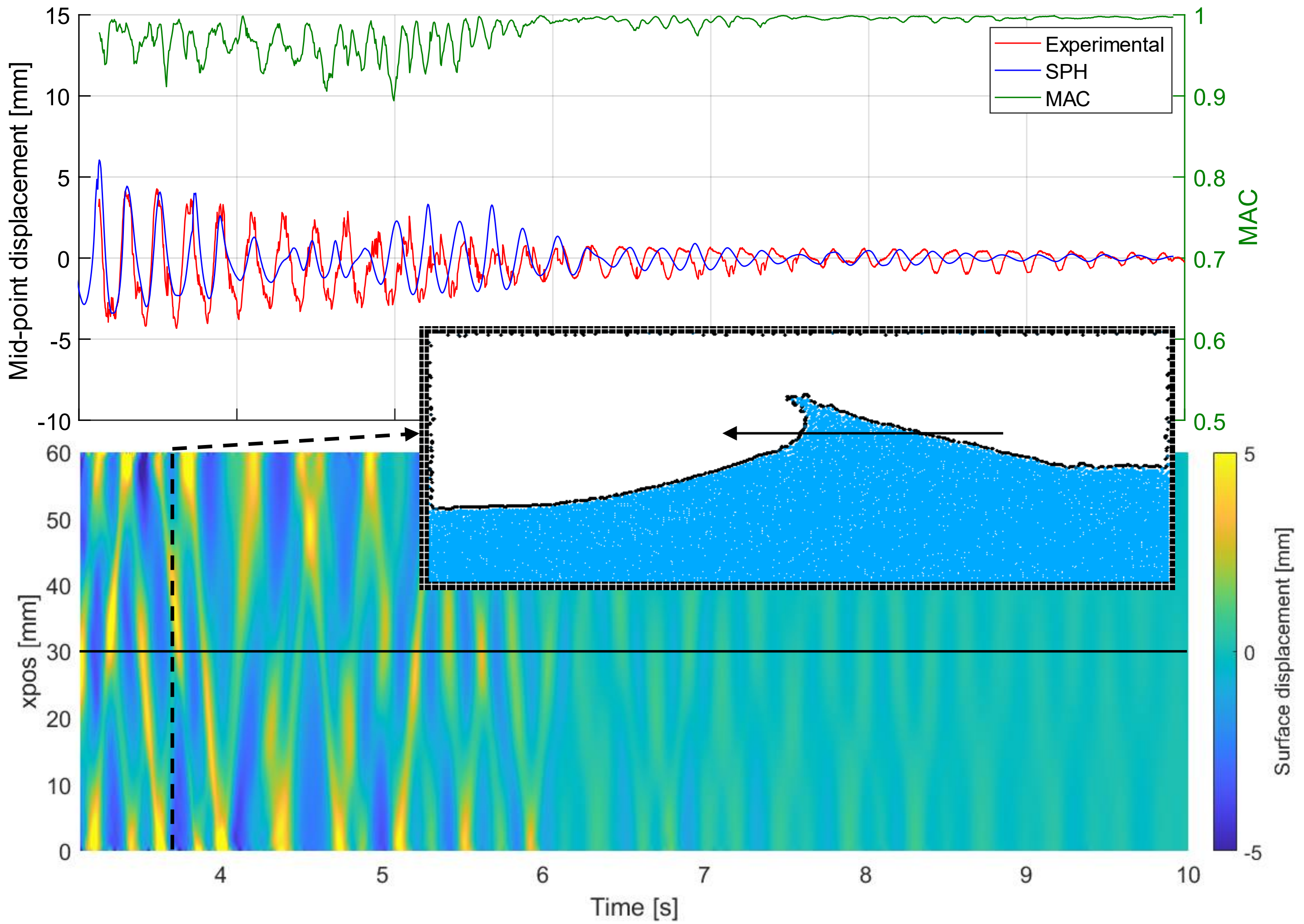

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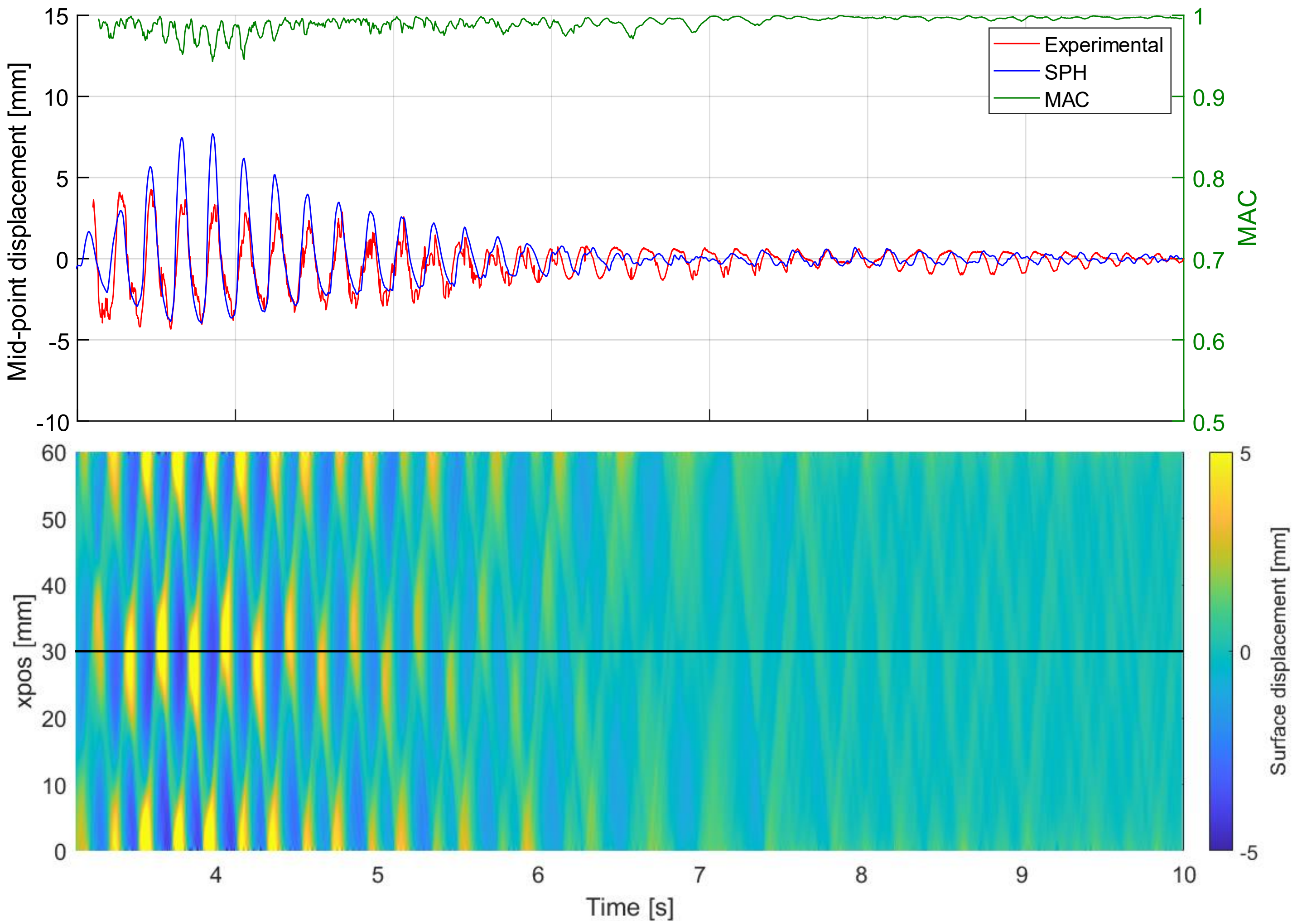

- Significant improvement with symmetric mode mostly present, improved correlation between surfaces.
- Asymmetric wave not directly driven by parametric excitation attenuated by surface tension.
- Correct fluid mode improves damping ratio, $\zeta_2 = 0.64\%$.
- Some discrepancy with experimental values. 3D wall effects likely drive coupled R2 motion.
- Important to resolve R2 damping accurately, a substantial energy dissipation mechanism.

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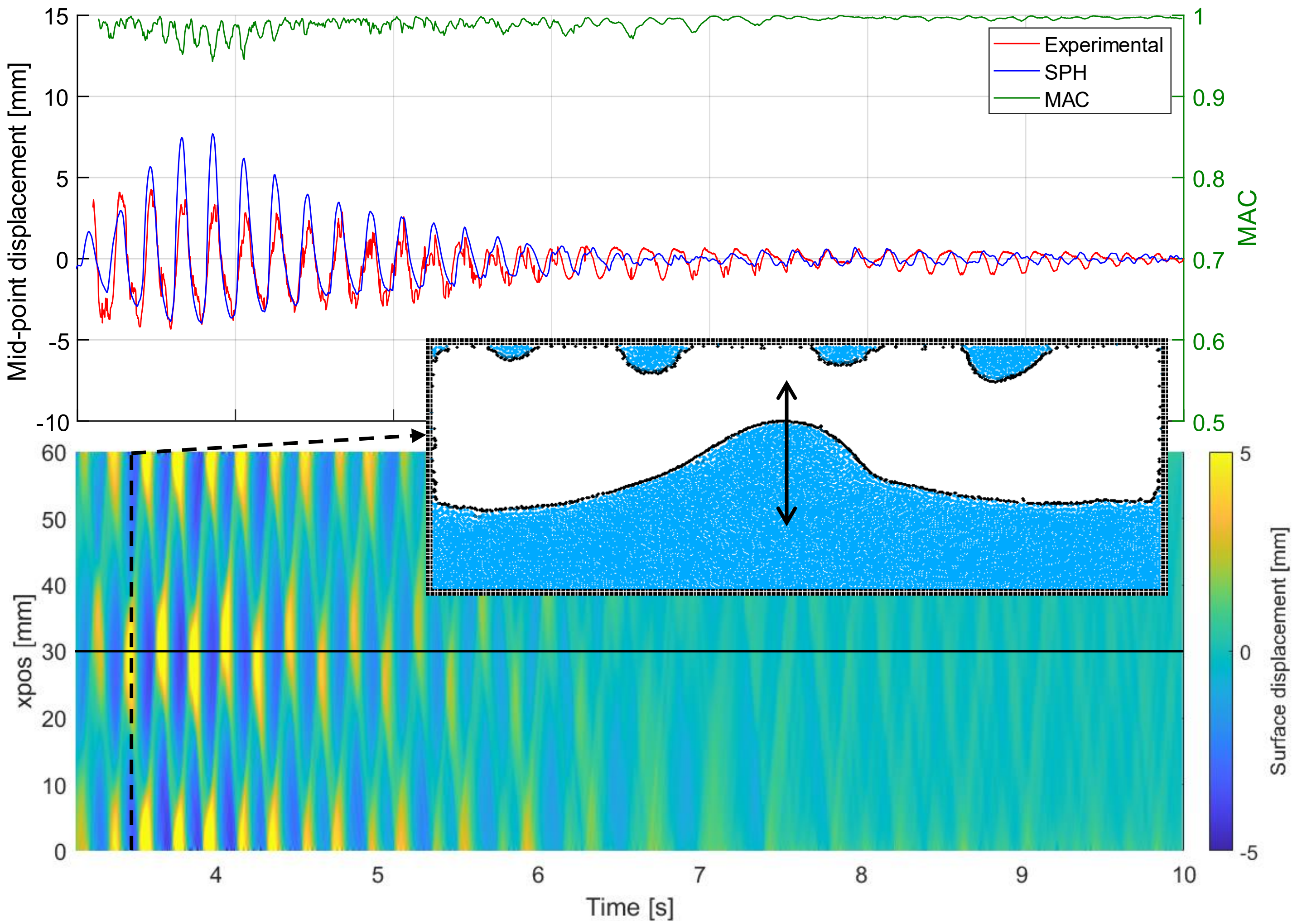

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Conclusions

- SDOF sloshing system experimentally studied
  - Pure vertical excitation, representing aircraft wing tank motion, studied to analyse corresponding fluid induced damping.
  - Three distinct damping regions observed, each with a characteristic flow regime.
  - R2 and R3 regime of focus, motion dominated by first symmetric sloshing mode.
- Experimental free surface motion identified
  - Technique developed for robust 2D surface identification with 3D parasitic influences.
  - Resulting surface response showed parametric resonance at the first symmetric mode, highlighting the important energy dissipation mechanisms within R2.
- Numerical modelling using SPH
  - Coupled to SDOF structural model and including a CSF surface tension model.
  - Presence of longitudinal waves deteriorates surface motion and SPH damping response. Surface tension appears to improve correlation.
  - Damping response still not correct, further development and calibration required.
- Future work at UoB continues to drive experimental and numerical sloshing studies.

Thank you!

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